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THESIS

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PERFORMANCE ANALYSIS OF
ALOHA NETWORKS UTILIZING
MULTIPLE SIGNAL POWER LEVELS

by

Randy L. Borchardt

June 1988

Thesis Advisor

Tri T. Ha

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PERFORMANCE ANALYSIS OF ALOHA NETWORKS
UTILIZING MULTIPLE SIGNAL POWER LEVELS

by

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B.S., Rensselaer Polytechnic Institute, 1978

Submitted in partial fulfillment of the
requirements for the degrees of

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ABSTRACT

This thesis develops the theory needed to determine the throughput and average packet transfer delay of both slotted and unslotted ALOHA networks utilizing multiple received signal power levels to create beneficial power capture effects in environments where near perfect capture does not occur. The throughput achievable can be greatly increased when two received power levels are utilized. Use of more than two equally spaced power levels provides no significant improvement in the throughput achievable when realistic capture thresholds are considered. The pseudo-Bayesian algorithm used to stabilize slotted ALOHA networks is theoretically adapted to systems employing two power levels.

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I. INTRODUCTION

A. GENERAL BACKGROUND INFORMATION

In network communications, information transferred between the members of the user population is typically formatted into discrete elements or logical divisions of the data, referred to as packets. Depending on the particular network, the packets may vary in length between a few bits and many thousand bits. With the rapid expansion in the size of the network user populations served and the geographical area over which they are distributed, packet radio broadcast systems have become popular in digital data communication networks. The broadcast capability of such systems allows reception of a signal transmitted over a common channel by all network nodes within range of the transmitter. Additionally, the radio channel provides a multiple access capability; that is, the channel may be simultaneously used by two or more stations within the network. When combined, these capabilities offer a great advantage in simplifying the topologies and routing of information necessary to interconnect all network nodes. The need for dedicated data links or circuit switching facilities to route information between users is effectively eliminated.

[Ref. 1:pp. 410-413]

Satellite communication systems provide an excellent example of the implementation of packet broadcast systems. Any number of stations may transmit signals up to the satellite on the uplink frequency, the multiple access channel. The satellite then retransmits the received signal back toward the earth on another frequency, the broadcast channel. This broadcasted signal may be received by all earth stations that are within the footprint of the satellite transmission beam. A network node retains the messages addressed to it, while discarding messages addressed to other stations. Since the downlink has only one transmitter dedicated to it, there are no conflicting traffic situations to be resolved. The problem that remains is how to achieve effective sharing of the multiple access channel among all users. [Ref. 1: pp. 411-413]

A variety of multiple access strategies are employed to realize effective channel use by all the network stations and maintain acceptable system performance. Performance of packet broadcast systems is typically measured by two parameters: the channel throughput S , defined as the average number of correctly received packet transmissions per packet transmission time or length, and the packet transfer delay TD , defined as the average time required to successfully transmit a packet to its destination. Conventional channel allocation schemes, such as frequency-division multiple-access (FDMA) and time-division

multiple-access (TDMA), effectively avoid the problems that arise when two or more sources attempt data transmission to a single destination by separating the signals in time or in the frequency spectrum. However, use of these techniques in many multiple access situations is inadequate or unwarranted due to other important considerations that include expandability, flexibility and simplicity of implementation. [Ref. 2:p. 401]

In digital data communications, the transmission requirements of the network users is highly variable. Traffic is typically generated in a bursty fashion; that is, data source duty cycles are low, and high peak-to-average ratios of the data rate are experienced. In interactive computer communication systems, for example, peak-to-average data rate ratios of 1000 to 1 are very common [Ref. 3:p. 362] and may be as high as 2000 to 1 [Ref. 1:p. 411]. Additionally, users that generate bursty traffic usually require their data to be successfully transmitted to the destination within specific delay constraints or require rapid acknowledgment of a successful transmission. [Ref. 3:pp. 352-353] Operation of a network under either of the FDMA or TDMA techniques, depending on the channel bit rate used, may result in extremely low utilization of the channel or introduce unacceptable large transfer delays. If a high channel bit rate is selected, transmission delays are small, but the channel remains idle

most of the time due to the low data source duty cycles. On the other hand, a low channel bit rate increases channel utilization but also increases the transfer delays experienced by the network users. [Ref. 1:p. 411]

To cope with some of these problems, the ALOHA random multiple access protocol was developed. Although no single technique can optimize system performance for all network characteristics, random multiple access techniques tend to be more efficient as the user population grows in number, shorter access delays are required, the traffic generation statistics become more bursty and user connectivity requirements become more demanding [Ref. 4:p. 703].

B. THE ALOHA RANDOM MULTIPLE ACCESS PROTOCOL

The random multiple access ALOHA protocol was first proposed at the University of Hawaii in 1970 to interconnect computers and terminals via radio and satellite channels. In an ALOHA system, it is assumed that a large number of users communicate with a central station or a common satellite over the same radio channel in uncoordinated manner. The users generate information according to a random process that leads to very bursty traffic statistics. For transmission, the data is formatted into fixed length packets that contain addressing information, parity check bits for error detection and any other required information. The common channel is

instantaneously available to any user that has a packet ready for transmission. Packet transmission can be made in relatively short bursts since the entire channel bandwidth is used. [Ref. 2:p. 401] Acknowledgments of accurately received packets, in the case of a terrestrial radio link, are broadcast by the central station over a side channel that can be made very reliable due to a very low data requirement [Ref. 5:p. 806]. In satellite broadcast communication networks, a station can receive its transmitted packet after the roundtrip propagation delay if the source station is within the satellite's footprint. If the transmitted packet is received without error, the station assumes that the destination also accurately received the packet and considers the transmission successful [Ref. 3:p. 362].

Packets from different sources will occasionally overlap at the receiver due to the independent, random generation of information at each network station. In this situation, a collision is said to have occurred at the receiver, and all packets involved are assumed to be destroyed. Upon reception of the garbled packet in a satellite network or receiving no acknowledgment from the central station in a terrestrial network, the affected terminals are considered to be backlogged and, in order to achieve reliable communications, repeatedly retransmit the collided packets until they are received correctly. To

prevent recurring collisions among the same users, retransmissions are attempted after random time intervals. New messages generated at a node attempting to resolve a collision are either lost to the system or stored in a buffer for later transmission. An ALOHA protocol is shown schematically in Figure 1. [Ref. 3:p. 362-363]

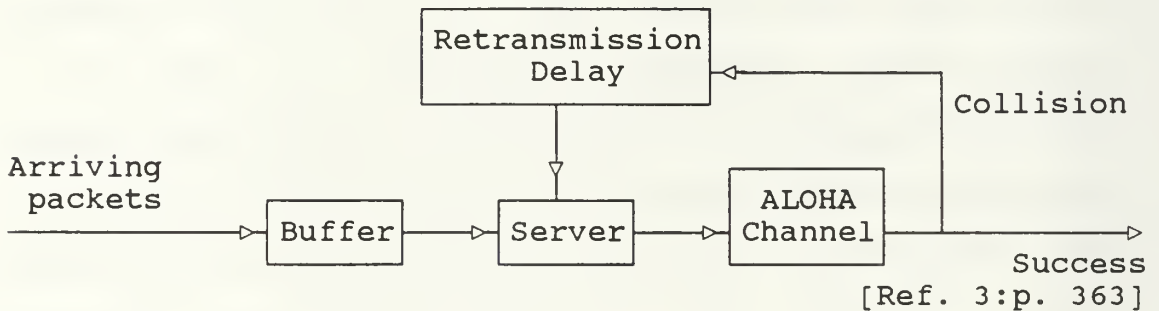


Figure 1.1: Representation of an ALOHA Protocol.

The costs of allowing all network users uncoordinated access to the channel are the collisions and subsequent retransmissions that take place. These factors limit the maximum throughput $S_{\max}$ and increase the packet transfer delays experienced. The ALOHA multiple access protocol outlined above, known as unslotted ALOHA, suffers from a low maximum throughput of $1/2e \approx 18.4$ percent. The throughput of unslotted ALOHA can be improved by coordinating the users' transmissions through control of packet arrival times at the receiver. Under this modification, known as slotted ALOHA, users are required to

synchronize the leading edges of their packet transmissions at the receiver with the start of a time slot having the same duration as a packet. No other modifications to the ALOHA scheme are made. The channel still remains available to each user that has a packet ready for transmission. The maximum throughput of a slotted ALOHA channel is $1/e \approx 36.8$ percent. [Ref. 3:pp. 366-368]

Whether slotted or unslotted, ALOHA systems are inherently unstable. They perform well in networks that are not heavily loaded or can maintain equilibrium between the throughput and the channel traffic rate; that is, the rate that newly generated and previously collided packets arrive at the network nodes for transmission is equal to the rate at which packets depart the system due to successful transmission. However, if fluctuations in the channel traffic rate occur such that the number of backlogged stations increases significantly, collisions happen more frequently, the channel becomes saturated, the throughput rapidly approaches zero and the packet transfer delays become unacceptably large. Very little is known about stabilization of unslotted ALOHA systems. On the other hand, various techniques have been proposed to prevent such failures from occurring in slotted ALOHA systems, and most involve adaptive control of the range of retransmission times or probabilities of retransmission of previously collided packets. [Ref. 6: pp. 215-217].

C. POWER CAPTURE IN ALOHA SYSTEMS

With the development of portable and mobile communications in urban environments and within physical structures such as large warehouses and the advent of very small aperture terminals (VSAT) in satellite communications, the ALOHA random multiple access protocol has received considerable interest in recent research. Much of this research has been directed toward improving throughput by considering power capture effects. If the receiver can accurately decode one of the packets involved in the collision, the successful packet is said to have captured the receiver in the presence of the interfering signals. Since the collision did not destroy all involved packets, the channel throughput will obviously increase. Power capture effects may occur naturally or be artificially created.

Natural power capture effects have been extensively studied [Refs. 4, 7-12]. These effects are present in two situations. The first arises when the network nodes are located at different distances from the receiver and no gain control is employed to equalize the power of the transmitted packets at the receiver. The power levels of the received packets may vary substantially due to spatial attenuation of the signals. This near/far phenomenon enhances the power capture capability of the receiver since the arriving packet with the highest power has the best

chance to be received accurately. [Ref. 5:p. 806]. The second situation arises when the channel subjects the transmitted packets to slow Rayleigh fading which creates different power classes among the received packets, again enhancing the power capture capability of the receiver [Ref. 7:p. 261].

Realizing the benefits of natural power capture effects when arriving packets have different power levels, it has been proposed that the power capture effect could be created in channels that do not experience fading by using multiple signal power levels for packet transmission [Refs. 11-16]. In these schemes, different users are assigned different power levels causing fixed priority among themselves or randomly select a power level for each transmission to avoid creating priority classes. As in natural capture, the packet with the highest power level has the best chance for successful reception. With the exception of References 11 and 12, all previous studies on created power capture effects are based upon slotted ALOHA systems that involve fixed priority classes among the users or allow near perfect capture to occur. Near perfect capture permits accurate reception of an arriving packet when the signal-to-interference ratio is between 0 dB and 3 dB. This is unrealistic in typical receivers since practical systems require signal-to-interference ratios between 6 dB and 12 dB to establish a usable range of

probabilities of error in the data packet, depending on the particular modulation scheme and coding technique employed. It has been shown that slotted ALOHA systems utilizing two random power levels for packet transmission achieve a maximum throughput rate of approximately 52 percent, while unslotted ALOHA systems attain a maximum throughput rate of slightly over 26 percent. [Refs. 11-12]

D. BASIC MODEL ASSUMPTIONS

To analyze the throughput S and the average packet transfer delay TD experienced in an ALOHA network, the following assumptions are widely accepted for use as a basis for the system model.

1. Infinite User Population

An infinitely large user population that collectively generates new data packets according to a Poisson process with parameter λ packets per packet length, the channel input rate, is assumed. Although this assumption appears to be invalid for realizable networks, it does provide a good approximation to a large, finite number of users that individually generate information packets rather infrequently [Ref. 17:p. 177]. For relatively small values of λ and low packet transfer delays, the number of backlogged nodes is typically insignificant, making the probability that a newly generated packet arrives at a backlogged node negligible.

Therefore, an infinite user population also ensures that the channel input rate λ does not fluctuate while stations await feedback concerning the success or failure of their transmissions since newly generated data packets can be assumed to arrive at idle nodes. [Ref. 6:pp. 210-211]

2. Poisson Channel Traffic

Since packets previously involved in collisions at the receiver require retransmission, the channel input rate does not accurately represent the true channel traffic rate imposed on the system by the users. If the average random retransmission delay is sufficiently large, the arrival of previously collided packets to the affected users can also be modeled as a Poisson process with parameter ϕ [Ref. 3: p. 363]. The combined newly generated and previously collided traffic, being the sum of two Poisson processes, can be modeled as another Poisson process with parameter $G = \lambda + \phi$ packets per packet length, the channel traffic rate. The probability that exactly k packets arrive at the network stations for transmission or at the receiver in an interval of t packet lengths is given by

$$\Pr\{k, t\} = \frac{(Gt)^k}{k!} \cdot \exp(-Gt) \quad (1.1)$$

3. Noise Free Channel

If a transmitted packet is able to capture the receiver, it will be decoded without error and the

transmission will be considered successful. Since the power levels used for transmission can be chosen to effectively negate errors due to channel background noise, only errors caused by collisions at the receiver are considered [Ref. 6:p. 210].

4. Negligible Processing Time

The processing time of the receivers required to decode the packets is negligible compared to the packet length and propagation delay. [Ref. 17:p. 292]

E. PURPOSE AND OUTLINE

The purpose of this thesis is to develop the theory needed to determine the throughput and packet transfer delay experienced in realistic slotted and unslotted ALOHA systems using multiple signal power levels to create the power capture effect in nonfading channels. In addition, the pseudo-Bayesian technique used to stabilize slotted ALOHA networks by changing the probability of packet transmission in a given slot based on an estimate of the number of backlogged stations will be adapted to systems using created capture effects.

Chapter II presents the detailed throughput and delay analysis of conventional (single received power level) unslotted ALOHA and then expands these results to the use of multiple power levels with realistic capture thresholds; that is, signal-to-interference ratios between 6 dB and

12 dB needed to produce a usable range for the probability of error in the data packet. Chapter III repeats the theme of Chapter II for slotted ALOHA networks. In Chapter IV, the pseudo-Bayesian stabilization technique used to prevent system failure in slotted ALOHA systems is discussed and adapted to multiple power level slotted ALOHA networks. Chapter V presents conclusions and recommendations for further research.

II. THROUGHPUT AND DELAY OF UNSLOTTED ALOHA NETWORKS

A. CONVENTIONAL UNSLOTTED ALOHA NETWORKS

In conventional unslotted ALOHA networks, users immediately begin transmission of newly generated packets regardless of the number of other stations currently utilizing the channel. Transmission power levels are assigned to each station such that packets are assumed to arrive at the receiver with equal powers after being spatially attenuated. Therefore, all users are equally successful in transferring data to the common receiver and created priority classes among the network stations are avoided. A collision occurs whenever two or more packets overlap even partially at the receiver. All messages involved in a collision are considered to be unusable and must be repeatedly retransmitted until successfully received.

To compute the throughput S achieved in an unslotted environment, the successful transmission of a reference message, referred to as the tagged packet, is considered. Assuming that messages have length τ and the tagged message arrives at the receiver at time t_0 , the tagged message will suffer a collision only if an interfering packet arrives at the receiver in the interval $(t_0 - \tau, t_0 + \tau)$ as shown in Figure 2.1. The probability that the tagged packet is

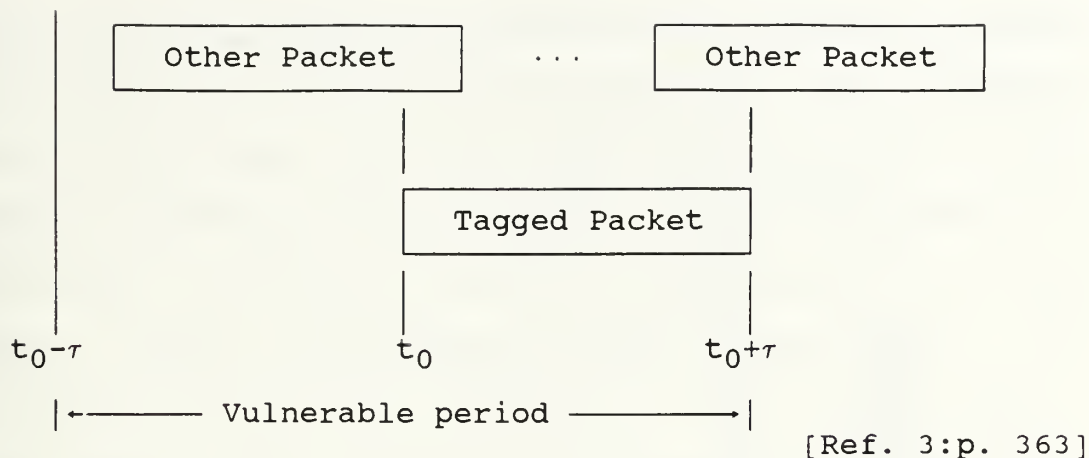


Figure 2.1: The Vulnerable Period for a Packet in Unslotted ALOHA Networks.

successfully transmitted is simply the probability that no other packets arrive at the receiver during its vulnerable period of two packet lengths. Thus from (1.1),

$$\begin{aligned} & \Pr\{\text{tagged packet successfully transmitted}\} \\ &= \Pr\{k=0, t=2\} = \exp(-2G) \end{aligned} \quad (2.1)$$

The throughput S is defined as the attempted channel traffic rate G multiplied by the probability that the tagged packet is successfully received; that is,

$$\begin{aligned} S &= G \cdot \Pr\{\text{tagged packet successfully transmitted}\} \\ &= G \cdot \exp(-2G) \end{aligned} \quad (2.2)$$

Figure 2.2 shows the channel throughput of conventional unslotted ALOHA systems versus the channel traffic rate. By differentiating (2.2) with respect to G , the maximum

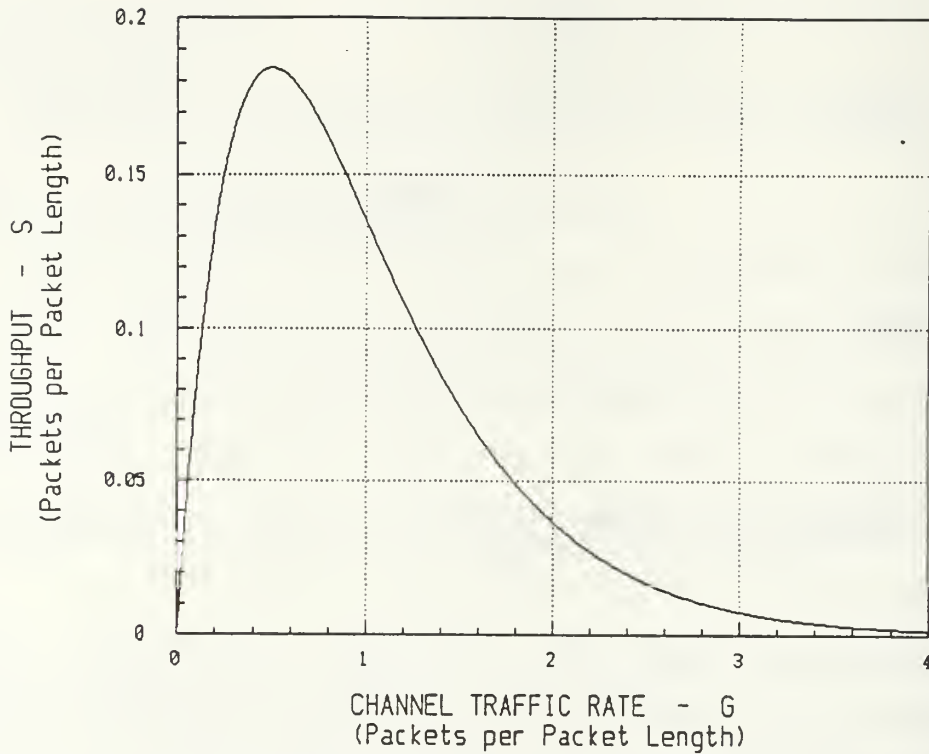


Figure 2.2: Throughput vs. Channel Traffic Rate in Conventional Unslotted ALOHA Networks.

throughput $S_{\max}$ is found to occur at $G = 0.5$ packets per packet length with a value

$$S_{\max} = \frac{1}{2e} = 0.184 \quad (2.3)$$

The relatively low throughput is a direct result of giving stations in the network uncoordinated access to the channel. [Ref. 3:pp. 362-364; Ref. 17:pp. 176-178].

The throughput results derived above assume steady-state conditions in the channel traffic rate. Close examination of Figure 2.2 shows that this may not always be

valid. If the traffic rate imposed on the system becomes larger than 0.5 packets per packet length, the throughput achieved decreases since the number of collisions at the receiver increases. This, in turn, causes an increase in the number of packets requiring retransmission and the channel traffic rate becomes larger. Consequently, the throughput suffers further reduction and a runaway effect takes place. This is the inherent unstable characteristic of ALOHA networks mentioned in Chapter I. Very little is known about stabilizing unslotted ALOHA systems other than operating the network at a throughput well below the maximum to allow sufficient margin for peak traffic demands. [Ref. 3:p. 364]

The packet transfer delay is composed of the packet length τ , the roundtrip propagation delay T_R and the average retransmission delay RD ; that is,

$$TD = T_R + \tau + RD \quad (2.4)$$

To determine the average packet transfer delay TD , the average retransmission delay RD is the only factor that needs to be determined since τ and T_R are known. As mentioned in Chapter I, the retransmission times are chosen randomly to prevent repeated collisions among the same users. Although various retransmission strategies exist, a uniform randomized retransmission strategy will be used because of its low cost and ease of implementation. Under

this strategy, the random time delay introduced after learning of the collision is uniformly distributed over 1 to K intervals of length τ and the average packet transmission delay is given by

$$RD = E\{r\} \cdot \left[T_R + \frac{(K + 1)\tau}{2} \right] \quad (2.5)$$

where $E\{r\}$ is the expected number of retransmissions required. Since the average number of attempts per successfully transmitted packet is G/S , the average number of retransmissions needed is one less; that is,

$$E\{r\} = \frac{G}{S} - 1 = \exp(2G) - 1 \quad (2.6)$$

Combining these results, the average packet transfer delay is found to be

$$TD = T_R + \tau + [\exp(2G) - 1] \cdot \left[T_R + \frac{(K + 1)\tau}{2} \right] \quad (2.7)$$

Figure 2.3 shows the average packet transfer delay, normalized to the packet length τ , versus the achieved throughput with K as a parameter. The propagation delay is neglected in the graph since it is dependent upon the particular network topology used and is negligible in some terrestrial applications. The inherent instability of ALOHA systems is again clearly demonstrated in Figure 2.3

since the average packet transfer delay rapidly approaches infinity for the throughput values corresponding to channel traffic rates above 0.5 packets per packet length. [Ref. 3: pp. 364-366; Ref. 17:pp. 178-181]

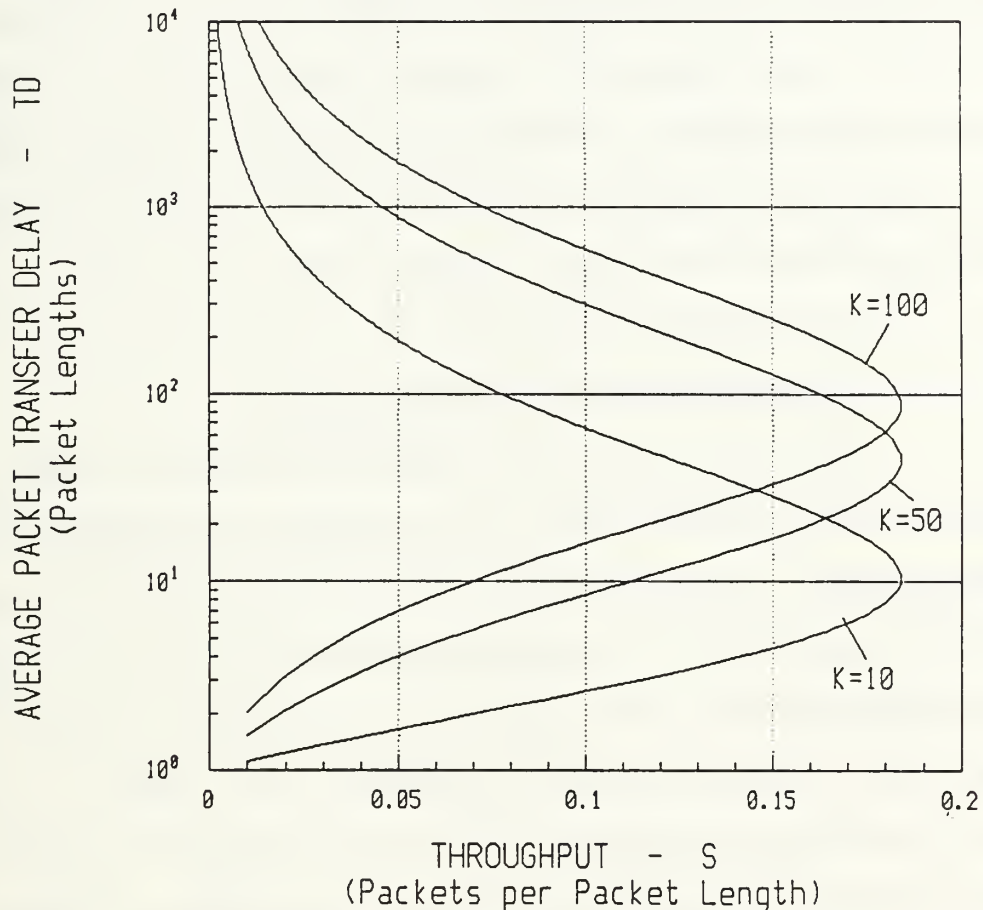


Figure 2.3: Average Packet Transfer Delay vs. Throughput in Conventional Unslotted ALOHA Networks.

B. CREATED POWER CAPTURE IN UNSLOTTED ALOHA NETWORKS

By using multiple power levels for packet transmission and reception, power capture effects can be created in

ALOHA networks to improve the throughput achieved and, therefore, decrease the average packet transfer delay experienced. With the use of multiple power levels, the tagged packet may capture the receiver and be received correctly in the presence of a number of other signals if the ratio of its power to the joint interference power exceeds a predetermined capture threshold γ_0 . Therefore, the system dynamics of unslotted ALOHA networks necessary to determine the statistics of the maximum number of interferers and their joint power in the interval $(t_0, t_0+\tau)$ are first discussed.

1. Unslotted ALOHA System Dynamics

This discussion on system dynamics is based on the channel model presented in Reference 18 to determine the throughput in unslotted code-division multiple-access (CDMA) systems. Although Reference 18 explores the throughput of a different multiple-access scheme, it is adaptable to the current discussion. The model is elegant in that it represents the stochastic process formed by the arrivals and departures of the interfering signals as binary numbers.

Assuming that k interfering packets are present at the receiver when the tagged packet arrives at time t_0 , referred to as "early interferers", there will be k departure events in the interval $(t_0, t_0+\tau)$ since each of the early interferers will complete transmission during

this time. The arrivals of the interfering packets are independent of one another and obey a Poisson process; therefore, the departure times are uniformly distributed over the interval $(t_0, t_0 + \tau)$. Additional interfering packets, referred to as "late interferers", will arrive at the receiver while the tagged packet is present and continue to be in the system for some random time after the tagged packet departs. Assuming that there are j independent late interferers, $k+j$ independent events will occur during the tagged packet's transmission at times uniformly distributed over the interval $(t_0, t_0 + \tau)$.

The $k+j$ arrival and departure events effectively partition the interval $(t_0, t_0 + \tau)$ into $k+j+1$ non-overlapping intervals of random length. If t_i for $1 \leq i \leq k+j$ is the time when the i^{th} event takes place, $I^{k,j}(t_i)$ will denote the number of interfering packets present at the receiver immediately after the occurrence of the i^{th} event. Obviously, $I^{k,j}(t_0)$ is equal to k and $I^{k,j}(t_{k+j})$ is equal to j . Each possible ordering of the $k+j$ events leads to a $(k+j+1)$ -vector realization, $r^{k,j} = \{I^{k,j}(t_0), I^{k,j}(t_1), \dots, I^{k,j}(t_{k+j})\}$, which uniquely determines the stochastic process $I^{k,j}(t)$ of interfering packets to which the tagged packet is subjected. Since the ordering of the events is arbitrary, there exist $c(k+j, k)$ equally likely realizations for the

number of interferers encountered by the tagged packet at the receiver, where the notation $c(x, y)$ is defined by

$$c(x, y) = \begin{cases} \frac{x!}{y! \cdot (x-y)!} & ; x, y, (x-y) \geq 0 \\ 0 & ; \text{otherwise} \end{cases} \quad (2.8)$$

For example, consider the situation where $k = 1$ and $j = 2$. The $c(3, 1) = 3$ possible realizations are given by

$$r^{1,2} = \{(1,0,1,2), (1,2,1,2), (1,2,3,2)\} \quad (2.9)$$

In each realization, there is one early interferer present at the receiver when the tagged packet arrives and two late interferers when the tagged packet departs; that is, $I^{1,2}(t_0) = 1$ and $I^{1,2}(t_3) = 2$. In the first realization, the early interferer ends its transmission before any late interferers arrive at the receiver as evidenced by $I^{1,2}(t_1) = 0$ and $I^{1,2}(t_2) = 1$. The evolution of $I^{1,2}(t)$ given by the first realization is illustrated in Figure 2.4. The other two realizations are evaluated similarly.

Reference 18 demonstrates that the probability of a realization $r^{k,j}$ depends only on the sum $n = k+j$ and not on k and j individually. As a result, there are exactly 2^n equally likely realizations of the n arrival and departure events given by an $(n+1)$ -vector r_n . Therefore, it is possible to put these n events in a one-to-one

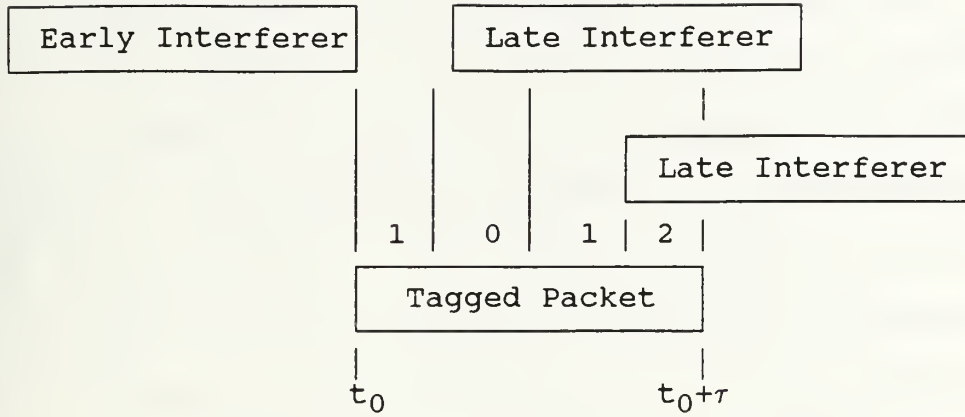


Figure 2.4: One Possible Realization of Three Interfering Packets.

correspondence with the n -bit binary numbers ranging from 0 to $2^n - 1$. In the discussion to follow, $r_{n,q}$ will refer to the $(n+1)$ -vector realization corresponding to the n -bit binary number representation of q . If the zeros of the binary number $(q)_2 = b_1 b_2 \dots b_n$, where $(q)_2$ is the binary representation of q where $0 \leq q \leq 2^n - 1$, represent the departures of early interferers and the ones represent the arrivals of late interferers, the number of interfering packets present at the receiver when the tagged packet arrives, $I_{n,q}(t_0)$, can be found by simply counting the number of zeros in the n -bit binary number $(q)_2$. The number of interfering packets present after the occurrence of the i^{th} event, $I_{n,q}(t_i)$ for $1 \leq i \leq n$, is computed according to the following recursive relation

$$I_{n,q}(t_i) = \begin{cases} I_{n,q}(t_{i-1}) - 1 & \text{if } b_i = 0 \\ I_{n,q}(t_{i-1}) + 1 & \text{if } b_i = 1 \end{cases} ; 1 \leq i \leq n \quad (2.10)$$

To find all possible 2^n realizations of n interfering signals encountered by the tagged packet, the n -bit binary numbers are listed from 0 to 2^n-1 , and the procedure outlined in the previous paragraph to determine the values of $I_{n,q}(t_i)$ for $0 \leq i \leq n$ is applied to each. As an example, the total ensemble of realizations for $n=3$ interfering packets with the associated three-bit binary representations is given in Table 2.1. Note that the realizations given in (2.9) are $r_{3,3}$, $r_{3,5}$ and $r_{3,6}$ respectively.

TABLE 2.1: POSSIBLE REALIZATIONS OF THREE INTERFERING PACKETS

q	$(q)_2$	$r_{3,q}$
0	000	(3,2,1,0)
1	001	(2,1,0,1)
2	010	(2,1,2,1)
3	011	(1,0,1,2)
4	100	(2,3,2,1)
5	101	(1,2,1,2)
6	110	(1,2,3,2)
7	111	(0,1,2,3)

2. Maximum Number Of Interferers Encountered

To derive the probability distribution function of the maximum number of interfering packets encountered by

the tagged packet if n other packets are known to interfere with its reception in a random fashion, a realization $r_{n,q} = \{I_{n,q}(t_0), I_{n,q}(t_1), \dots, I_{n,q}(t_n)\}$ with its associated n -bit binary representation $(q)_2 = b_1b_2\dots b_n$ is chosen at random. Obviously, the maximum number of interferers is given by the maximum value of $I_{n,q}(t_i)$ where $0 \leq i \leq n$.

As mentioned earlier, it is assumed that the zero bits in $(q)_2$ represent the departure of early interferers and the one bits represent the arrival of late interferers in the realization. The probability that $(q)_2$ will contain exactly ℓ zeros or, equivalently, the probability that exactly ℓ early interferers (EI) are present at the receiver when the tagged packet arrives is given by

$$\begin{aligned} \Pr(\text{number of zeros in } (q)_2 = \ell | n) \\ = \Pr\{EI = \ell | n\} = 2^{-n} \cdot c(n, \ell) \end{aligned} \quad (2.11)$$

Conditioning on the number of early interferers, the probability distribution function of the maximum number of interferers is given by

$$\begin{aligned} \Pr(\max(I_{n,q}(t)) = j | n) \\ = \sum_{\ell=0}^n \Pr(\max(I_{n,q}(t)) = j | EI = \ell, n) \cdot \Pr\{EI = \ell | n\} \\ = 2^{-n} \cdot \sum_{\ell=0}^n \Pr(\max(I_{n,q}(t)) = j | EI = \ell, n) \cdot c(n, \ell) \end{aligned} \quad (2.12)$$

To assist the evaluation of the conditional probability that appears in the summation of (2.12), a symmetric random Bernoulli random variable and a symmetric Bernoulli random walk will be defined on the n bits of $(q)_2$. Since zeros and ones in $(q)_2$ occur with equal probability, a symmetric Bernoulli random variable Y can therefore be defined on the ensemble $\varepsilon_Y = \{-1, 1\}$ by

$$Y_i = \begin{cases} 1 & \text{if } b_i = 1 \\ -1 & \text{if } b_i = 0 \end{cases} ; i = 1, 2, \dots, n \quad (2.13)$$

A symmetric Bernoulli random walk V can be generated by the sums of the independent symmetric Bernoulli random variables Y_i ; that is,

$$\begin{aligned} V(i) &= Y_1 + Y_2 + \dots + Y_i \\ &= V(i-1) + Y_i \quad ; 1 \leq i \leq n \end{aligned} \quad (2.14)$$

where, by definition, $V(0) = 0$. [Ref. 19:p. 208]

With the definition of the symmetric random walk in (2.14), the stochastic process $I_{n,q}(t_i)$ for $1 \leq i \leq n$ can be redefined in terms of $V(i)$ as

$$I_{n,q}(t_i) = I_{n,q}(t_0) + V(i) = \ell + V(i) \quad (2.15)$$

Applying the maximum operator to (2.15), the maximum number of packets interfering with the reception of the tagged packet at the receiver can be related to the maximum value of the symmetric Bernoulli random walk by the expression

$$\max(V) = \max(I_{n,q}(t)) - \ell = j - \ell \quad (2.16)$$

The conditioning event of (2.12), $EI = \ell$, is equivalent to the condition that $V(n) = n - 2\ell$, that is, the number of late interferers minus the number of early interferers. Combining this fact with (2.16), the conditional probability in (2.12) can be expressed as

$$\begin{aligned} \Pr\{\max(I_{n,q}(t)) = j | EI = \ell, n\} \\ &= \Pr\{\max(V) = j - \ell | V(n) = n - 2\ell, n\} \\ &= \frac{\Pr\{(\max(V) = j - \ell) \text{ and } (V(n) = n - 2\ell) | n\}}{\Pr\{V(n) = n - 2\ell | n\}} \end{aligned} \quad (2.17)$$

The denominator of (2.17), as a consequence of the reflection principle in random walk probability theory, can be evaluated as [Ref. 20:p. 75]

$$\Pr\{V(n) = n - 2\ell | n\} = 2^{-n} \cdot c(n, n - \ell) \quad (2.18)$$

Since the maximum number of interferers j is always greater than or equal to the number of late interferers $n - \ell$, $n - 2\ell \leq j - \ell$ and the probability that the symmetric random walk at epoch n has a value $V(n) = n - 2\ell$ and $\max(V) \geq j - \ell$ is

$$\begin{aligned} \Pr\{V(n) = n - 2\ell \text{ and } \max(V) \geq j - \ell | n\} \\ &= \Pr\{V(n) = 2(j - \ell) - (n - 2\ell) = 2j - n | n\} \\ &= 2^{-n} \cdot c(n, j) \end{aligned} \quad (2.19)$$

With these conditions, the probability that $\max(V) = j - \ell$ and $V(n) = n - 2\ell$ is the difference between (2.19) and the similar expression evaluated for $\max(V) \geq j - \ell + 1$; that is,

$$\begin{aligned}
& \Pr\{\max(V) = j - \ell \text{ and } V(n) = n - 2\ell | n\} \\
&= \Pr\{V(n) = n - 2\ell \text{ and } \max(V) \geq j - \ell | n\} \\
&\quad - \Pr\{V(n) = n - 2\ell \text{ and } \max(V) \geq j - \ell + 1 | n\} \\
&= \Pr\{V(n) = 2j - n | n\} - \Pr\{V(n) = 2j + 2 - n | n\} \\
&= 2^{-n} \cdot [c(n, j) - c(n, j+1)] \tag{2.20}
\end{aligned}$$

Note that this is the numerator of (2.17). [Ref. 20: pp. 89]

Substituting (2.18) and (2.20) into (2.17), the probability distribution of the maximum number of interferers encountered by the tagged packet during the interval $(t_0, t_0 + \tau)$ conditioned on the number of early interferers is given by

$$\begin{aligned}
& \Pr\{\max(I_{n,q}(t)) = j | EI = \ell, n\} \\
&= \frac{[c(n, j) - c(n, j+1)]}{c(n, n - \ell)} \tag{2.21}
\end{aligned}$$

Recall that in the derivation of (2.19), the condition $j \geq n - \ell$ was mentioned. This is equivalent to $\ell \geq n - j$. The other condition on ℓ that has been implied throughout the preceding discussion is $\ell \leq j$, that is, the maximum number of interferers is always greater than or equal to the number of early interferers. Substituting

(2.21) and these conditions into (2.12), the probability density function of the maximum number of interferers encountered when n other packets are known to partially overlap the tagged packet at the receiver is given by

$$\begin{aligned}
 & \Pr\{\max(I_{n,q}(t)) = j | n\} \\
 &= 2^{-n} \cdot \sum_{\ell=n-j}^j \frac{[c(n,j) - c(n,j+1)]}{c(n,n-\ell)} \cdot c(n,\ell) \\
 &= \begin{cases} 2^{-n} \cdot c(n,j) \cdot \left[\frac{(2j - n + 1)^2}{j + 1} \right] & ; n \geq j \geq \left\lceil \frac{n}{2} \right\rceil \\ 0 & ; \text{otherwise} \end{cases} \quad (2.22)
 \end{aligned}$$

where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x .

The number of realizations $r_{n,q}$ in which the maximum number of interferers equals j , denoted by $C_j(n)$, can be found by simply multiplying the probability density function for $\max(I_{n,q}(t))$ by 2^n ; that is,

$$\begin{aligned}
 & C_j(n) = 2^n \cdot \Pr\{\max(I_{n,q}(t)) = j | n\} \\
 &= \begin{cases} c(n,j) \cdot \left[\frac{(2j - n + 1)^2}{j + 1} \right] & ; n \geq j \geq \left\lceil \frac{n}{2} \right\rceil \\ 0 & ; \text{otherwise} \end{cases} \quad (2.23)
 \end{aligned}$$

Table 2.2 gives a list of $C_j(n)$ for values of n and j ranging from one to six.

TABLE 2.2: NUMBER OF REALIZATIONS r_n WITH A
MAXIMUM OF j INTERFERERS - $C_j(n)$

n	$C_1(n)$	$C_2(n)$	$C_3(n)$	$C_4(n)$	$C_5(n)$	$C_6(n)$
1	2	0	0	0	0	0
2	1	3	0	0	0	0
3	0	4	4	0	0	0
4	0	2	9	5	0	0
5	0	0	10	16	6	0
6	0	0	5	27	25	7

3. Unslotted ALOHA With Two Power Levels

In ALOHA networks utilizing two power levels to create power capture effects, data packets from all stations arrive at the receiver after being spatially attenuated with one of two normalized power levels given by the set $\Omega = \{1, M\}$. It is assumed that each user randomly selects a received power level for each data packet from Ω according to some probabilistic rule common to all so that every user in the network has an equal chance to successfully transmit information [Ref. 16:p. 1026]. The higher power level M is chosen according to the relation

$$(N + 1) \cdot \gamma_0 > M \geq N \cdot \gamma_0 \quad (2.24)$$

where $N \geq 1$ is an integer and γ_0 is the power capture threshold of the receiver. Recall that γ_0 is between 6 dB and 12 dB for the systems of interest.

The tagged packet, arriving at the receiver with power $P_t \in \{1, M\}$, may capture the receiver and be successfully transmitted given a realization $r_{n,q}$ of n early and late interferers, each with a power P_j , if and only if

$$P_t \geq \gamma_0 \cdot \max \left[\begin{array}{c} I_{n,q}(t_i) \\ \sum_{j=1} P_j \end{array} \right] \quad ; \quad 0 \leq i \leq n \quad (2.25)$$

To determine the throughput of a ALOHA network utilizing two received signal power levels, the probability that the tagged packet is successfully transmitted must be conditioned on the number of interfering packets encountered at the receiver. From (2.25) and the assumption of a noise free channel, the tagged packet will obviously be successful if no interfering packets are present during the interval $(t_0, t_0 + \tau)$. Therefore, the channel throughput S is

$$\begin{aligned} S &= G \cdot \Pr\{\text{tagged packet successfully transmitted}\} \\ &= G \cdot \sum_{n=0}^m \Pr\{\text{tagged packet successful} | n\} \cdot \Pr\{n\} \\ &= G \cdot \exp(-2G) \cdot \left[1 + \sum_{n=1}^m \Pr\{\text{capture} | n\} \cdot \frac{(2G)^n}{n!} \right] \end{aligned} \quad (2.26)$$

where n is the number of interfering packets present in the interval $(t_0, t_0 + \tau)$, m is the maximum number of interferers

the tagged packet can tolerate during its transmission and $\Pr\{n\}$ is given by (1.1) with t equal to two packet lengths.

For the tagged packet to capture the receiver in the presence of $n \geq 1$ interfering packets, two events must occur. The tagged packet must have the received power level M while all interfering packets have the lower power level 1. Letting this be event A ,

$$\Pr\{A|n\} = 2^{-(n+1)} \quad (2.27)$$

This event modifies (2.25) to

$$P_t = M \geq N \cdot \gamma_0 \geq \gamma_0 \cdot \left[\begin{array}{c} \max(I_{n,q}(t)) \\ \sum_{j=1} \quad P_j \end{array} \right] = \gamma_0 \cdot \max(I_{n,q}(t)) \quad (2.28)$$

where the definition of M from (2.24) has been included. Obviously, the maximum number of interferers that the tagged packet can tolerate at any instant in time is N . Therefore, the second event B that must occur is

$$N \geq \max(I_{n,q}(t)) \quad (2.29)$$

Each of these N interferers can presumably be early interferers, finish their transmission and then transmit newly generated data packets during the interval $(t_0, t_0 + \tau)$. Therefore, the maximum number of interferers that the tagged packet can tolerate throughout its transmission period is $m = 2N$. Noting directly from (2.23)

$$\sum_{j=0}^N \Pr\{\max(I_{n,q}(t)) = j|n\} = 2^{-n} \cdot \sum_{j=0}^N C_j(n) \quad (2.31)$$

the probability of event B occurring when n other packets are known to interfere with the tagged packet is

$$\Pr\{B|n\} = \begin{cases} 2^{-n} \cdot \sum_{j=0}^N C_j(n) & ; n \leq 2N \\ 0 & ; \text{otherwise} \end{cases} \quad (2.30)$$

Since events A and B are independent, the probability that the tagged packet captures the receiver when n other packets interfere with its accurate reception is given by the product of the conditional probabilities of events A and B; that is,

$$\begin{aligned} \Pr\{\text{capture}|n\} &= \Pr\{A|n\} \cdot \Pr\{B|n\} \\ &= \begin{cases} 2^{-(2n+1)} \cdot \sum_{j=0}^N C_j(n) & ; n \leq 2N \\ 0 & ; \text{otherwise} \end{cases} \end{aligned} \quad (2.31)$$

Substituting (2.31) into (2.26) yields the system throughput of the two power level model {1, M}

$$S = G \cdot \exp(-2G) \cdot \left[1 + \sum_{n=1}^{2N} 2^{-(2n+1)} \cdot \frac{(2G)^n}{n!} \cdot \left[\sum_{j=0}^N C_j(n) \right] \right] \quad (2.32)$$

where $C_j(n)$ is given by (2.23). The throughput of an unslotted ALOHA network employing two power levels is

plotted in Figure 2.5 for various values of N . Observing that throughput increases with larger values of N , an ideal upper limit on the throughput achievable can be found by letting $N \rightarrow \infty$ in (2.32), giving

$$\begin{aligned}
 S_{\infty} &= G \cdot \exp(-2G) \cdot \left[1 + \sum_{n=1}^{\infty} 2^{-(2n+1)} \cdot \frac{(2G)^n}{n!} \cdot \left[\sum_{j=0}^{\infty} c_j(n) \right] \right] \\
 &= \frac{G \cdot \exp(-2G)}{2} \cdot [1 + \exp(G)] \quad (2.34)
 \end{aligned}$$

This is also plotted in Figure 2.5. The maximum throughput achievable is seen to increase from $S_{\max} \approx 0.24$ at $G \approx 0.64$ with $N = 1$ to $S_{\max} \approx 0.26$ at $G \approx 0.76$ as $N \rightarrow \infty$. Therefore, the system throughput and the channel traffic rate that can be tolerated is greatly improved by utilizing two received power levels. However, no significant improvement is obtained for values of N larger than three. Note that the throughput can be represented in terms of M and γ_0 by substituting

$$N = \left\lfloor \frac{M}{\gamma_0} \right\rfloor \quad (2.33)$$

where the notation $\lfloor x \rfloor$ represents the greatest integer less than or equal to x .

The average packet transfer delay in unslotted ALOHA networks utilizing two received power levels is

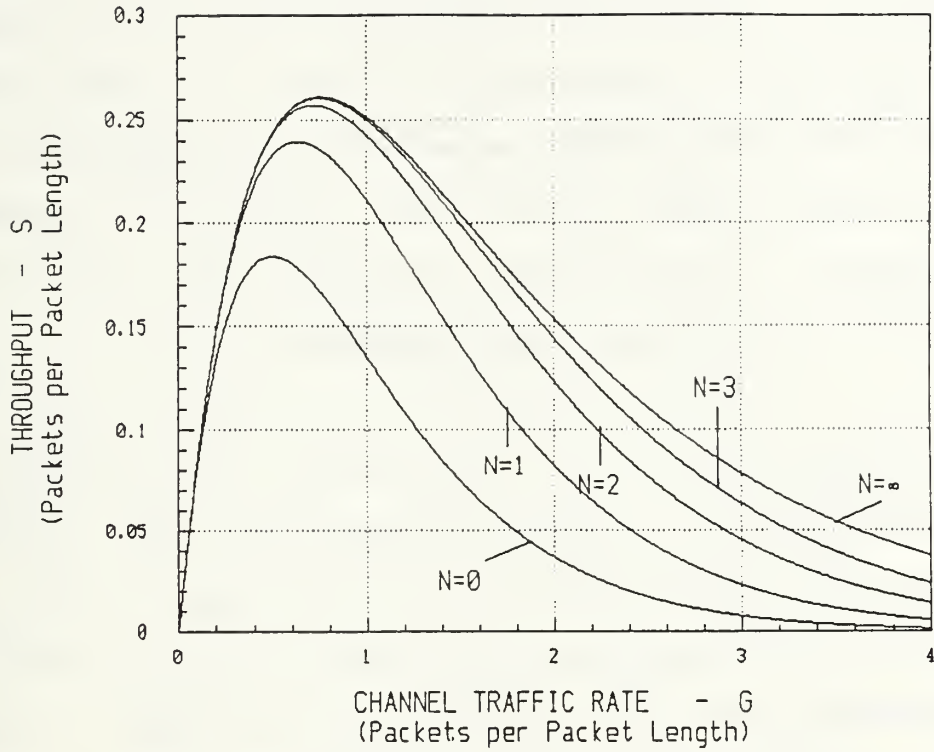


Figure 2.5: Throughput vs. Channel Traffic Rate in Unslotted ALOHA Networks Utilizing Two Power Levels. $\Omega = \{1, (N+1)\gamma_0 > M \geq N\gamma_0\}$. ($N = 0$ corresponds to conventional unslotted ALOHA).

analyzed in the same fashion as conventional unslotted ALOHA networks. Thus, the average packet transfer delay is

$$TD = T_R + \tau + \left[\frac{G}{S} - 1 \right] \cdot \left[T_R + \frac{(K+1)\tau}{2} \right] \quad (2.35)$$

where S is given by (2.32). The average packet transfer delay, normalized to a packet length, versus throughput rate achieved for an unslotted ALOHA network utilizing the two received signal power levels $\Omega = \{1, 3\gamma_0\}$ is shown in

Figure 2.6. Again, the propagation delay T_R is assumed to be negligible. Comparison of Figure 2.6 with Figure 2.3 shows that the average packet delay in systems utilizing two received power levels, as a result of the improved throughput, is considerably less than that experienced in conventional unslotted ALOHA systems.

Although use of two received signal power levels has shown to be advantageous in unslotted ALOHA systems, Figures 2.5 and 2.6 demonstrate that the inherent instability characteristic of ALOHA systems still exists; that is, as the value of the offered channel traffic rate increases beyond that giving maximum throughput, throughput approaches zero and average packet transfer delay increases towards infinity. Although the network must still operate at throughput values well below the maximum, use of two received signal power levels allows operation at higher throughput rates with greater margin for peak traffic demands.

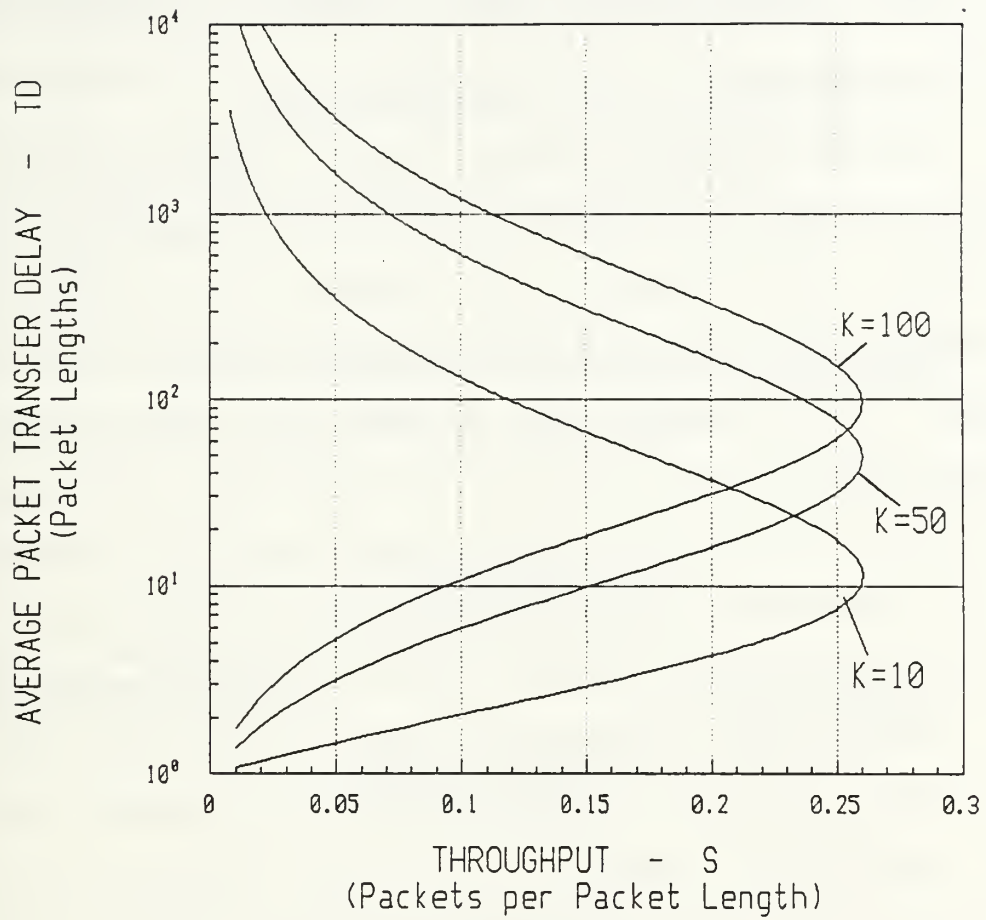


Figure 2.6: Average Packet Transfer Delay vs. Throughput in Unslotted ALOHA Networks Utilizing Two Signal Power Levels. $\Omega = \{1, 3\gamma_0\}$.

III. THROUGHPUT AND DELAY OF SLOTTED ALOHA NETWORKS

A. CONVENTIONAL SLOTTED ALOHA NETWORKS

In slotted ALOHA systems, time is segmented into slots having a duration equal to the packet transmission time or length. User stations are required to synchronize the transmissions of their data packets so that the leading edges of the packets are aligned with the beginning of predetermined time slots at the receiver. To accomplish this synchronization, a data packet ready for transmission at an arbitrary time must be delayed until the start of the next slot after its arrival before transmission begins. As in conventional unslotted ALOHA, users are assigned transmission power levels such that all packets arrive at the receiver with equal power to avoid inadvertent creation of priority classes among the users and give all stations equal opportunity to communicate with the receiver. Collisions at the receiver are characterized by complete overlap and destruction of all involved packets. Affected packets require repeated retransmission until successfully received. [Ref. 17:pp. 286-287]

Since packet arrivals at the receiver are synchronized, the number of interfering packets will remain constant over the interval $(t_0, t_0 + \tau)$. Therefore, the vulnerable period experienced by the tagged packet is reduced to one packet

length of duration τ as shown in Figure 3.1. Since the probability that the tagged packet is successfully transmitted equals the probability that no other packets are present during the tagged packet's vulnerable period of one packet length given by (1.1), the throughput S of slotted ALOHA systems can be related to the channel traffic rate G by

$$\begin{aligned} S &= G \cdot \Pr\{\text{tagged packet successfully transmitted}\} \\ &= G \cdot \Pr\{k = 0, t = 1\} = G \cdot \exp(-G) \end{aligned} \quad (3.1)$$

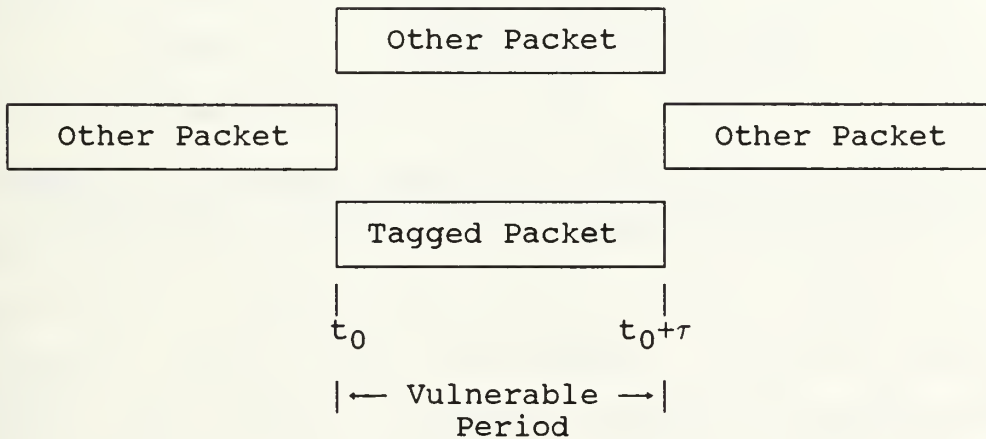


Figure 3.1: The Vulnerable Period for a Packet in Slotted ALOHA Networks.

The maximum throughput $S_{\max}$ of slotted ALOHA systems, found by differentiating (3.1) with respect to G , occurs at $G = 1$ and has a value

$$S_{\max} = \frac{1}{e} = 0.368 \quad (3.2)$$

which is twice that of conventional unslotted ALOHA. Figure 3.2 illustrates the improvement in throughput of conventional slotted ALOHA systems over unslotted ALOHA systems as a function of the channel traffic rate. [Ref. 3:pp. 366-368; Ref. 17:pp. 287-289]

The analysis of packet transfer delay TD in slotted systems is similar to that used for unslotted ALOHA. However, the random time delay introduced awaiting the start of the next slot for transmission must be accounted for in the derivation. Due to the Poisson arrival process, network users generate packets at times uniformly distributed over an interval $(0, \tau)$, where τ is the duration of a slot. Thus, the average waiting time is simply $\tau/2$. Since the roundtrip propagation delay is rarely equal to an integral multiple of the packet length, this average waiting time also applies to previously collided packets. Using the same uniform randomized retransmission strategy introduced in the previous chapter to avoid repeated collisions among the same group of users, the average packet transfer delay TD in slotted ALOHA systems is

$$\begin{aligned}
 TD &= T_R + \tau + \frac{\tau}{2} + E\{r\} \cdot \left[T_R + \frac{(K+1)\tau}{2} + \frac{\tau}{2} \right] \\
 &= T_R + \frac{3\tau}{2} + \left[\frac{G}{S} - 1 \right] \cdot \left[T_R + \frac{(K+2)\tau}{2} \right] \quad (3.3)
 \end{aligned}$$

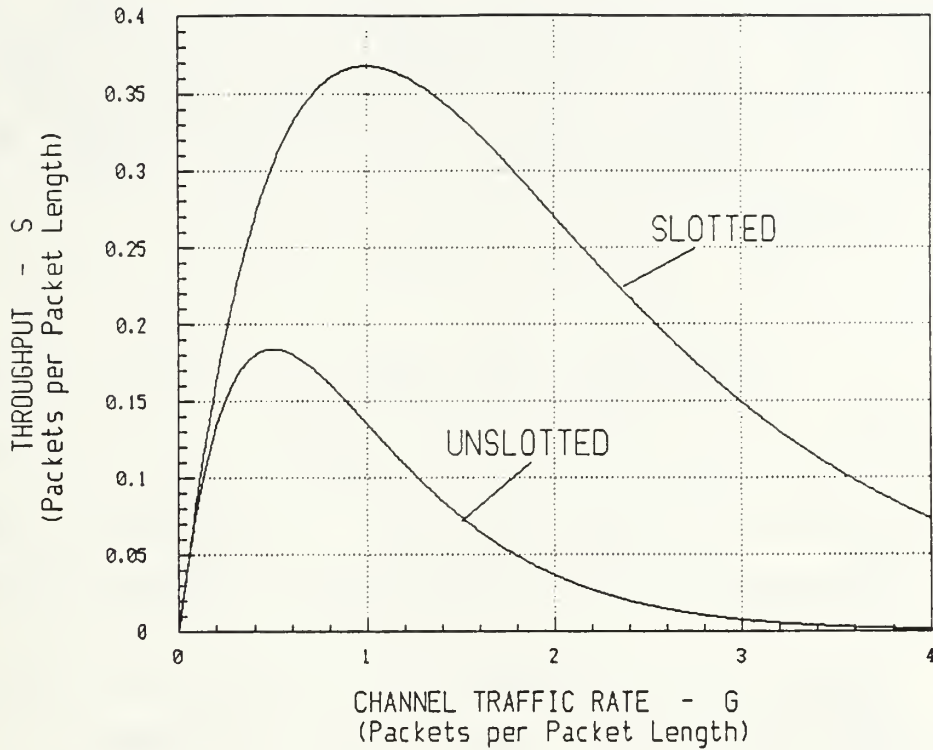


Figure 3.2: Throughput vs. Channel Traffic Rate in Conventional Slotted and Unslotted ALOHA Networks.

where T_R is the roundtrip propagation delay and K is the maximum time delay in packet lengths introduced before retransmission after learning of a collision. [Réf. 3: p. 369] Although a more detailed and accurate analysis of the average packet transfer delay exists to better account for the time delay caused by collisions at the receiver through the use of a Markov model of the system, (3.3) has been shown to be a good approximation for values of $K \gg 1$. Figure 3.3 shows the average packet transfer delay, normalized to the packet length τ , versus the achieved

throughput with K as a parameter for conventional slotted ALOHA networks in which the roundtrip propagation delay is assumed to be negligible. [Ref. 3:pp. 366-378; Ref. 17: pp. 287-299]

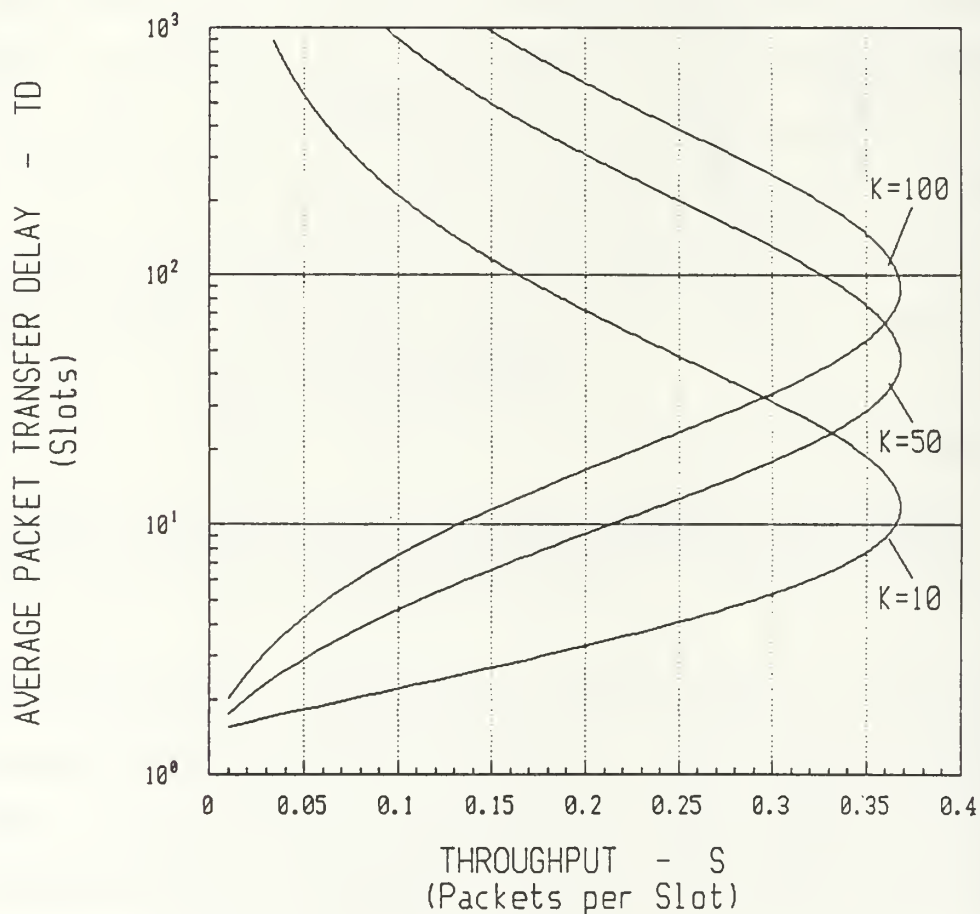


Figure 3.3: Average Packet Transfer Delay vs. Throughput in Conventional Slotted ALOHA Networks.

Although slotted ALOHA systems demonstrate better throughput and packet transfer delay performance than unslotted systems, Figures 3.2 and 3.3 reveal that these

systems retain the inherent instability characteristic exhibited in unslotted ALOHA networks. As the channel traffic rate increases beyond 1.0 packets per packet length, the throughput is reduced and the average packet transfer delay increases towards infinity. Various methods have been proposed to solve this problem by dynamically adjusting the range of retransmission times or probabilities of retransmission of previously collided packets. In Chapter IV, the pseudo-Bayesian technique used to stabilize slotted ALOHA systems will be discussed.

B. CREATED POWER CAPTURE IN SLOTTED ALOHA NETWORKS

As in unslotted ALOHA systems, power capture effects can be created by employing multiple signal power levels. Data packets from all stations are assumed to arrive at the receiver after spatial attenuation with one of the multiple power levels contained in the set Ω . Each user randomly chooses one of the equally likely received power levels from Ω for each packet transmitted or retransmitted. Therefore, the equal opportunity for each station to successfully communicate with the receiver is preserved, and priority classes among the users are avoided. [Ref. 16:p. 1026]

The receiver will be able to capture and successfully decode the tagged packet if the ratio of the tagged packet's power to the joint power of the interfering

packets exceeds the capture threshold γ_0 of the receiver, where γ_0 is between 6 dB and 12 dB for systems of interest. Since the number of interferers n is constant over the interval $(t_0, t_0+\tau)$, capture occurs if and only if

$$P_t \geq \gamma_0 \cdot \sum_{j=1}^n P_j \quad (3.4)$$

where P_t is the power of the tagged packet and P_j is the power of an interfering packet. Two models employing multiple received power levels will be considered.

1. Slotted ALOHA With Two Power Levels

In the first model to be considered, two received signal power levels are utilized to create the desired power capture effects; that is, $\Omega = \{1, M\}$. The higher power level M is chosen according to the relation

$$(N + 1) \cdot \gamma_0 > M \geq N \cdot \gamma_0 \quad (3.5)$$

where $N \geq 1$ is an integer. Obviously from (3.4) and (3.5), the maximum number of interferers that the tagged packet can tolerate during the interval $(t_0, t_0+\tau)$ is given by

$$\max(n) = \begin{cases} N = \left\lfloor \frac{M}{\gamma_0} \right\rfloor & ; P_t = M \\ 0 & ; P_t = 1 \end{cases} \quad (3.6)$$

Conditioning on the number of interferers present during the interval $(t_0, t_0 + \tau)$, the system throughput can be expressed as

$$\begin{aligned}
 S &= G \cdot \Pr\{\text{tagged packet successfully transmitted}\} \\
 &= G \cdot \sum_{n=0}^N \Pr\{\text{capture}|n\} \cdot \Pr\{n\} \\
 &= G \cdot \exp(-G) \cdot \left[1 + \sum_{n=1}^N \Pr\{\text{capture}|n\} \cdot \frac{G^n}{n!} \right] \quad (3.7)
 \end{aligned}$$

where $\Pr\{n\}$ is given by (1.1) and $\Pr\{\text{capture}|n = 0\} = 1$. For the receiver to capture the tagged packet, the tagged packet must have power $P_t = M$ and all of the n interferers must have power $P_j = 1$. Thus, the probability of capture conditioned on the number of interferers n is given by

$$\Pr\{\text{capture}|n\} = \frac{1}{2} \cdot \frac{1}{2^n} = \frac{1}{2^{n+1}} \quad (3.8)$$

Substituting (3.8) into (3.7), the throughput of a slotted ALOHA system utilizing two received signal power levels, $\Omega = \{1, M\}$, is

$$S = G \cdot \exp(-G) \cdot \left[1 + \frac{1}{2} \cdot \sum_{n=1}^N \frac{(G/2)^n}{n!} \right] \quad (3.9)$$

Since throughput is observed to increase with increasing values of N , an ideal upper limit on the

throughput achievable can be found by letting $N \rightarrow \infty$ in (3.9) and is given by

$$\begin{aligned}
 S &= G \cdot \exp(-G) \cdot \left[1 + \frac{1}{2} \cdot \sum_{n=1}^{\infty} \frac{(G/2)^n}{n!} \right] \\
 &= \frac{G \cdot \exp(-G)}{2} \cdot [1 + \exp(G/2)] \quad (3.10)
 \end{aligned}$$

The throughput of slotted ALOHA networks employing two received power levels is plotted in Figure 3.4 for various values of N . The maximum throughput is seen to increase from $S_{\max} \approx 0.47$ at $G \approx 1.23$ with $N = 1$ to $S_{\max} \approx 0.52$ at $G \approx 1.5$ as $N \rightarrow \infty$. Therefore, the system throughput and channel traffic rate that can be tolerated is greatly improved by using two received signal power levels. Note that with $N = 6$, the throughput is extremely close to that achieved as $N \rightarrow \infty$. No significant improvement is gained for values of N larger than six.

Reference 16 gives similar results to those obtained above. However, in Reference 16, the upper limit in the equation corresponding to (3.9) is $N = M-1$. Substituting this value of N into (3.5) gives

$$\frac{M}{M-1} \cdot \gamma_0 > \frac{M}{M-1} \geq \gamma_0 \quad (3.11)$$

Therefore, the model used in Reference 16 allows near perfect capture to occur since the ratio of the tagged packet's power to the joint interference power is less than 3 dB and approaches 0 dB with increasing values of M or N. This is contrary to practice where realistic thresholds are between 6 dB and 12 db.

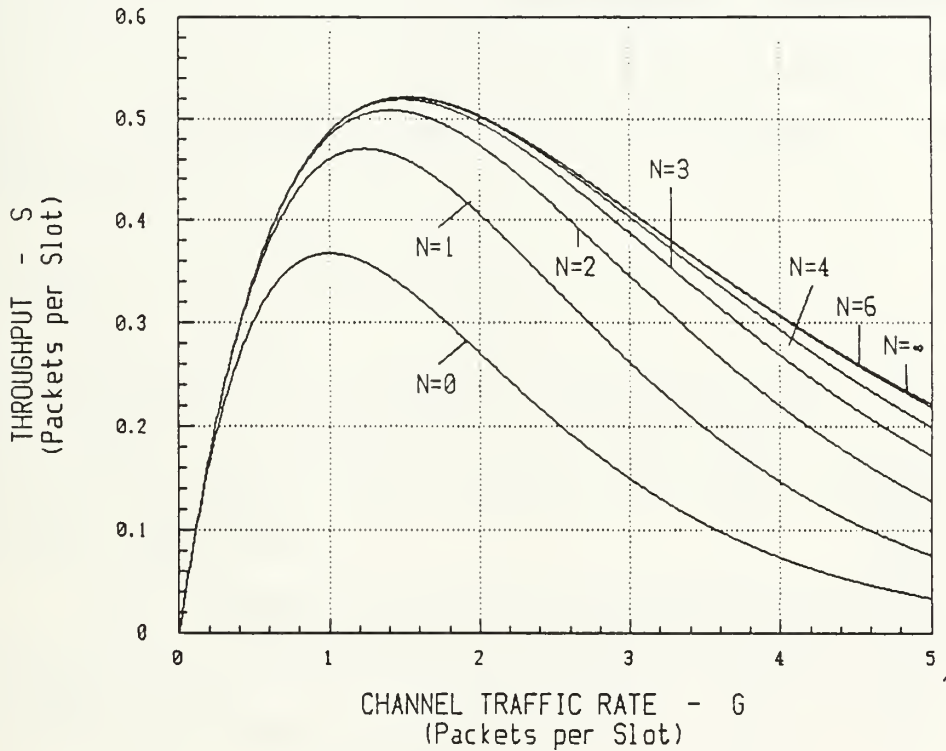


Figure 3.4: Throughput vs. Channel Traffic Rate in Slotted ALOHA Networks Utilizing Two Power Levels. $\Omega = \{1, (N+1)\gamma_0 > M \geq N\gamma_0\}$. (N = 0 corresponds to conventional slotted ALOHA.)

The average packet transfer delay in slotted ALOHA networks employing two received signal power levels is

obtained by substituting the expression for throughput in (3.9) into (3.3). Figure 3.5 shows the average packet transfer delay, normalized to the packet length, versus the throughput achieved with K as a parameter for slotted ALOHA networks using $\Omega = \{1, 6\gamma_0\}$, where K is the maximum delay in packet lengths introduced before retransmission after learning of a collision. The propagation delay T_R is considered to be negligible.

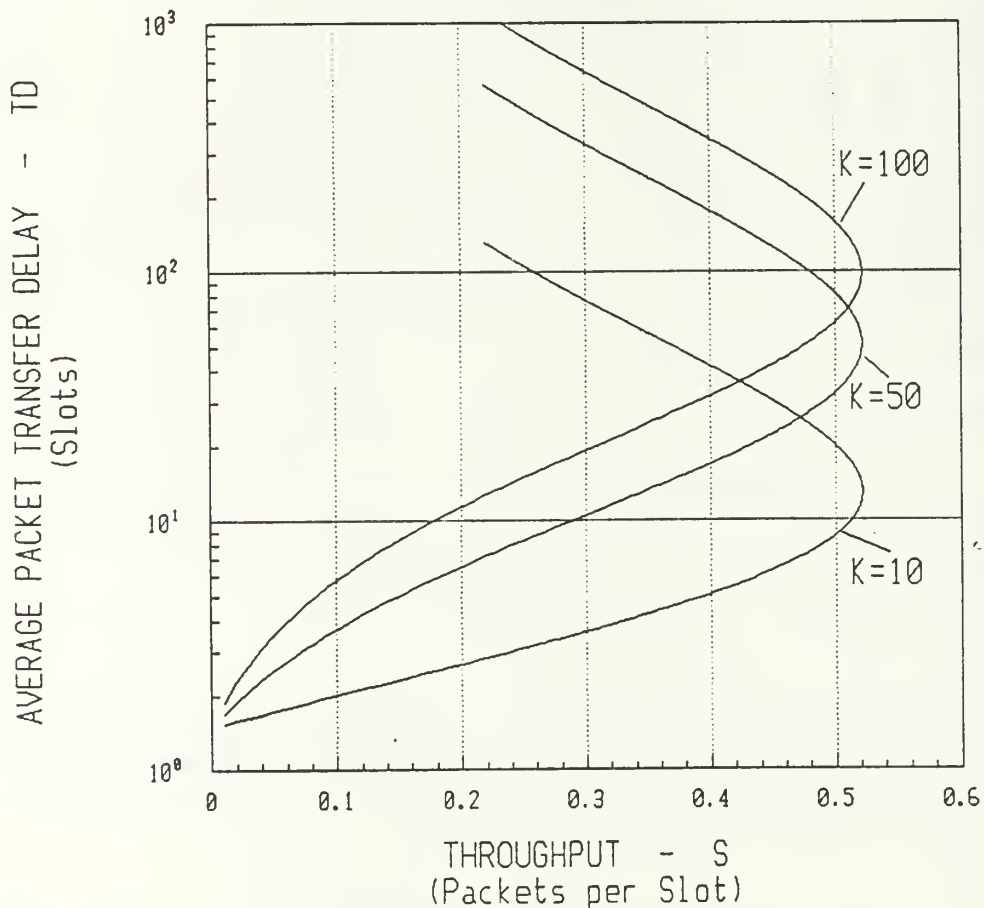


Figure 3.5: Average Packet Transfer Delay vs. Throughput in Slotted ALOHA Networks Utilizing Two Signal Power Levels. $\Omega = \{1, 6\gamma_0\}$.

Figures 3.4 and 3.5 clearly demonstrate that the inherent instability of ALOHA networks remains in systems utilizing two received signal power levels. The pseudo-Bayesian stabilization technique will be adapted to these systems in Chapter IV.

2. Slotted ALOHA With Multiple Power Levels

The second model to be considered employs the set of equally likely received signal power levels $\Omega = \{1, 2, \dots, M\}$ to create the desired power capture effects. The highest power level M is chosen to satisfy the relation

$$N = \left\lfloor \frac{M}{\gamma_0} \right\rfloor \quad (3.12)$$

where N is the maximum number of interferers that can be tolerated by the tagged packet during the interval $(t_0, t_0 + \tau)$ if $P_t = M$.

The throughput of a slotted ALOHA system operating with this set of received power levels is again given by (3.7), repeated here for convenience

$$\begin{aligned} S &= G \cdot \Pr\{\text{tagged packet successfully transmitted}\} \\ &= G \cdot \sum_{n=0}^N \Pr\{\text{capture} | n\} \cdot \Pr\{n\} \\ &= G \cdot \exp(-G) \cdot \left[1 + \sum_{n=1}^N \Pr\{\text{capture} | n\} \cdot \frac{G^n}{n!} \right] \end{aligned} \quad (3.7)$$

If $P_t = m \in \{1, 2, \dots, M\}$ and n other packets are known to interfere with the reception of the tagged packet, the minimum value m can assume, such that (3.4) is satisfied, is given by

$$m \geq \lceil n \cdot \gamma_0 \rceil \quad (3.13)$$

where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x . Combining (3.4) and (3.13), $\Pr\{\text{capture}|n\}$ is the probability that $P_t = m$ and the probability that the joint power of the n interferers is less than or equal to m/γ_0 , summed over all possible values of m ; that is,

$$\Pr\{\text{capture}|n\} = \sum_{m=\lceil n\gamma_0 \rceil}^M \Pr\{P_t=m\} \cdot \Pr\left\{\sum_{j=1}^n P_j \leq \left\lfloor \frac{m}{\gamma_0} \right\rfloor\right\} \quad (3.14)$$

Since the power levels in $\Omega = \{1, 2, \dots, M\}$ have a uniform probability distribution, the method of generating functions yields [Ref. 20:pp. 284-285]

$$\Pr\left\{\sum_{j=1}^n P_j \leq \left\lfloor \frac{m}{\gamma_0} \right\rfloor\right\} = M^{-n} \cdot \sum_{k=0}^{\infty} (-1)^k \cdot c(n,k) \cdot c\left[\left\lfloor \frac{m}{\gamma_0} \right\rfloor - Mk, n\right] \quad (3.15)$$

The summation in the above equation exists only for $k = 0$ due to the definition of $c(x,y)$. Therefore,

$$\Pr\left\{\sum_{j=1}^n P_j \leq \left\lfloor \frac{m}{\gamma_0} \right\rfloor\right\} = M^{-n} \cdot c\left[\left\lfloor \frac{m}{\gamma_0} \right\rfloor, n\right] \quad (3.16)$$

Substituting (3.16) into (3.14) gives $\Pr\{\text{capture}|n\}$ as [Ref. 20:p. 64]

$$\begin{aligned}\Pr\{\text{capture}|n\} &= \sum_{m=\lceil n\gamma_0 \rceil}^M M^{-(n+1)} \cdot c\left(\left\lfloor \frac{m}{\gamma_0} \right\rfloor, n\right) \\ &= M^{-(n+1)} \cdot c\left(\left\lfloor \frac{M}{\gamma_0} \right\rfloor + 1, n+1\right)\end{aligned}\quad (3.17)$$

Hence the channel throughput for a slotted ALOHA network utilizing the set of multiple received signal power levels $\Omega = \{1, 2, \dots, M\}$ is given by

$$S = G \cdot \exp(-G) \cdot \left[1 + \sum_{n=1}^{\lfloor M/\gamma_0 \rfloor} M^{-(n+1)} \cdot c\left(\left\lfloor \frac{M}{\gamma_0} \right\rfloor + 1, n+1\right) \cdot \frac{G^n}{n!} \right] \quad (3.18)$$

This expression for the throughput with $\gamma_0 = 1+\delta$, where δ is a small positive number, is identical to that derived in Reference 16. As in the two power level model, the upper limit of the summation in the throughput equation of Reference 16 is $N = M-1$, thus allowing near perfect capture to occur for increasing values of N or M . This is again contrary to practice where realistic capture thresholds are between 6 dB and 12 dB.

Figure 3.6 shows the throughput of slotted ALOHA systems utilizing multiple received signal power levels $\Omega = \{1, 2, \dots, 20\}$ with γ_0 equal to 0 dB and 6 dB. If near perfect capture occurs, the throughput is greatly

improved as evidenced by $S_{\max} \approx 0.63$ at $G \approx 1.6$ packets per packet length. Reference 16 demonstrates that this is near the ideal upper limit on the throughput achievable as $M \rightarrow \infty$. However, when the capture threshold is changed to the minimum realistic value of 6 dB, the maximum throughput drops to $S_{\max} \approx 0.38$ at $G \approx 1.0$ packets per packet length. As a result, the average packet transfer delay will be approximately that of conventional ALOHA. Therefore, use of multiple received power levels in slotted ALOHA networks provides no advantage over the two power level model when realistic capture thresholds are considered.

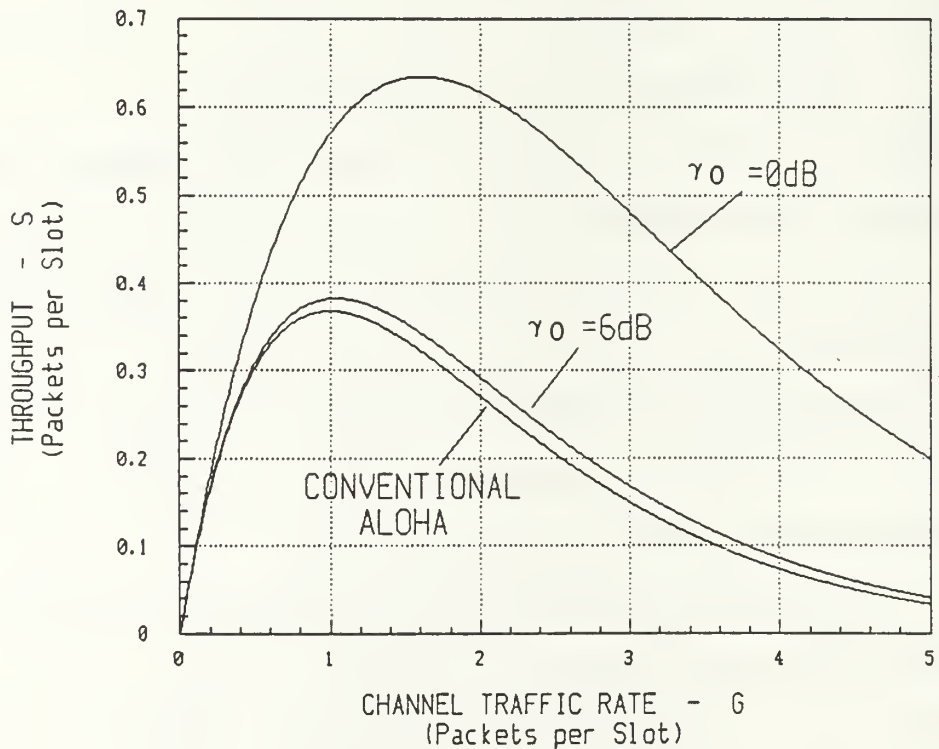


Figure 3.6: Throughput vs. Channel Traffic Rate in Slotted ALOHA Networks Utilizing Multiple Power Levels. $\Omega = \{1, 2, \dots, 20\}$.

IV. PSEUDO-BAYESIAN STABILIZATION OF SLOTTED ALOHA

A. STABILIZATION OF CONVENTIONAL SLOTTED ALOHA

The pseudo-Bayesian algorithm provides a simple and effective way to stabilize ALOHA networks at the maximum throughput achievable and prevents the severe degradation of system performance when the number of backlogged stations increases significantly due to fluctuations in the channel traffic rate. This algorithm differs from conventional slotted ALOHA in two ways. First, newly generated packets are regarded as backlogged immediately on arrival at the transmitting station and treated in the same manner as previously collided packets. Secondly, rather than using the uniform retransmission strategy employed in the preceding chapters, all network users determine a packet broadcast probability q_r for each slot based on an estimate of the total number of backlogged stations in the network. If a station has a data packet ready for transmission, it transmits the packet in the slot with probability q_r , independent of any previous attempts to transmit the packet or the time that the station has been backlogged. Note that the concept of a tagged packet will not be used in the following discussion and the user population will be treated as a whole. Although this requires minor modification to some of the theory developed

in Chapter III, the results previously presented remain valid. [Ref. 21: pp. 1-2; Ref. 6:p. 217]

Just prior to the beginning of a slot, each station with a data packet ready for transmission must decide whether or not to transmit its packet. Obviously, each slot has three possible outcomes dependent on the number of users that attempt to access the channel. These are:

- Idle - no stations transmit in the slot.
- Success - only one user transmits in the slot.
- Collision - more than one user transmits and no packet is successfully received.

When an idle or collision slot occurs, no stations receive any feedback other than the fact that an idle or collision slot occurred. All stations are informed of a success slot and the identification of the user that transmitted the successfully received packet. To dynamically change the estimate of the number of backlogged stations waiting to transmit data and the broadcast probability q_r for subsequent slots, each station considers only the network idle/success/collision history gained from this limited feedback. Thus, all network stations should compute the same value of q_r for each slot. [Ref. 21:pp. 1-2]

The pseudo-Bayesian stabilization algorithm assumes that, at the beginning of a slot k , there are n backlogged stations in the network waiting to transmit data. This includes all new data packets generated prior to the

beginning of the slot. The value of n is assumed to be a Poisson random variable with mean v . The value of v represents the users' estimate of the number of backlogged stations in the network. Thus, the probability distribution function of n is [Ref. 21:p. 6]

$$\Pr\{n\} = \frac{\exp(-v) \cdot v^n}{n!} \quad (4.1)$$

Since each station computes the same value for q_r , the attempted channel traffic for a slot k will be

$$G_k = n \cdot q_r \quad (4.2)$$

The probability that a successful transmission will occur (a success slot S) is the probability that only one of the n backlogged stations transmit while the other $n-1$ users continue to wait. Thus,

$$\Pr\{\text{success slot} = S | n\} = n \cdot q_r \cdot (1 - q_r)^{n-1} \quad (4.3)$$

Averaging over the ensemble of possible values for the number of backlogged users if a successful transmission occurs, the expected probability of a success slot is

$$\begin{aligned} \Pr\{S\} &= \sum_{n=1}^{\infty} \Pr\{S | n\} \cdot \Pr\{n\} \\ &= \sum_{n=1}^{\infty} n \cdot q_r \cdot (1 - q_r)^{n-1} \cdot \frac{\exp(-v) \cdot v^n}{n!} \\ &= v \cdot q_r \cdot \exp(-v \cdot q_r) \end{aligned} \quad (4.4)$$

To maximize the probability of a success slot, the optimal broadcast probability q_r is easily computed. Since q_r is a probability and must be less than or equal to one,

$$q_r = \min\left\{1, \frac{1}{v}\right\} \quad (4.5)$$

Note that (4.4) is identical to the expression for throughput of a slotted ALOHA network given by (3.1). Therefore, the broadcast probability given in (4.5) attempts to maintain a channel traffic rate $G = 1$ for each slot and the channel throughput at its maximum value. All that remains is development of the method to update the estimated number of backlogged stations. [Ref. 21:p. 6; Ref. 6:p. 218]

Bayes Rule will be utilized to update the estimate of the expected number v of backlogged stations in the network for subsequent slots and to find the probability distribution of n after receiving feedback from the present slot; that is,

$$\Pr\{n|E\} = \frac{\Pr\{E|n\} \cdot \Pr\{n\}}{\Pr\{E\}} \quad (4.6)$$

where E represents the outcome (idle, success or collision) of the present slot. Throughout the process, the Poisson assumption on the number of data packets in the system will be preserved. As will be shown, an approximation to the

probability of n given a collision occurred in the previous slot must be made to preserve the Poisson distribution of backlogged packets. For this reason, the algorithm is referred to as pseudo-Bayesian. [Ref. 21:p. 4-7]

Assuming that the number n of backlogged stations in the network at a given time is a Poisson random variable with mean $v \geq 1$ and that each transmits its data packet with probability $q_r = 1/v$, the probability that an idle slot (I) occurs is

$$\Pr\{I|n\} = \left[1 - \frac{1}{v}\right]^n \quad (4.7)$$

The expected probability of an idle slot is determined by averaging over n ; that is,

$$\begin{aligned} \Pr\{I\} &= \sum_{n=0}^{\infty} \Pr\{I|n\} \cdot \Pr\{n\} \\ &= \sum_{n=0}^{\infty} \left[1 - \frac{1}{v}\right]^n \cdot \frac{\exp(-v) \cdot v^n}{n!} \\ &= \exp(-v) \cdot \sum_{n=0}^{\infty} \frac{(v-1)^n}{n!} = \exp(-1) \end{aligned} \quad (4.8)$$

Application of Bayes Rule (4.6) yields

$$\Pr\{n|I\} = \frac{\Pr\{I|n\} \cdot \Pr\{n\}}{\Pr\{I\}} = \frac{\exp(-v+1) \cdot (v-1)^n}{n!} \quad (4.9)$$

Therefore, the number n of backlogged stations has a Poisson distribution with mean $\max\{v-1,0\}$. When an idle slot occurs, the stations reduce their estimate of the expected number of backlogged stations by one. If v is already less than one, v is set to zero. [Ref. 21:p. 7]

Applying Bayes Rule to (4.1), (4.3) and (4.4), the probability distribution of n after a success slot is

$$\Pr\{n|S\} = \frac{\Pr\{S|n\} \cdot \Pr\{n\}}{\Pr\{S\}} = \frac{\exp(-v+1) \cdot (v-1)^{n-1}}{(n-1)!} \quad (4.10)$$

The term $n-1$ in the right side of (4.10) reflects the departure of a successful packet from the system. The resulting distribution is again Poisson with mean $v-1$. Therefore, the network users decrement their estimate of the expected number of backlogged stations by one after learning of a success slot. [Ref. 21:p. 7]

The probability that a collision slot occurs is equal to the probability that two or more of the backlogged stations attempt transmission in the slot; that is,

$$\begin{aligned} \Pr\{C|n\} &= \sum_{m=2}^n c(n,m) \cdot \left[\frac{1}{v}\right]^m \cdot \left[1 - \frac{1}{v}\right]^{n-m} \\ &= 1 - \sum_{m=0}^1 c(n,m) \cdot \left[\frac{1}{v}\right]^m \cdot \left[1 - \frac{1}{v}\right]^{n-m} \\ &= 1 - \left[1 - \frac{1}{v}\right]^n - n \cdot \left[\frac{1}{v}\right] \cdot \left[1 - \frac{1}{v}\right]^{n-1} \end{aligned} \quad (4.11)$$

where $c(n,m)$ is defined by (2.8). Since only three outcomes are possible for each slot, the expected probability of a collision slot (C) is

$$\Pr\{C\} = 1 - \Pr\{I\} - \Pr\{S\} = 1 - 2 \cdot \exp(-1) \quad (4.12)$$

Applying Bayes Rule to (4.1), (4.11) and (4.12) yields the probability distribution of n given a collision slot as

$$\begin{aligned} \Pr\{n|C\} &= \frac{\Pr\{C|n\} \cdot \Pr\{n\}}{\Pr\{C\}} \\ &= \frac{\exp(-v+1)}{n! \cdot [\exp(1)-2]} \cdot [v^n - (v-1)^{n-1} \cdot (v-1+n)] \quad (4.13) \end{aligned}$$

This distribution is shown in Figure 4.1 for various values of v . Although (4.13) is not a Poisson distribution, $\Pr\{n|C\}$ can be closely approximated by a Poisson distribution with mean $v+[\exp(1)-2]^{-1}$; that is,

$$\Pr\{n|C\} = \frac{\exp[v+(\exp(1)-2)^{-1}]}{n!} \cdot [v+(\exp(1)-2)^{-1}]^n \quad (4.14)$$

Figure 4.2 plots the distributions given by (4.13) and (4.14). As seen from Figure 4.2, the Poisson approximation to the actual distribution is rather good and improves with increasing values of v . Therefore, when a collision occurs, the estimate of the expected number of backlogged stations is increased by $[\exp(1)-2]^{-1}$. [Ref. 21:p. 7; Ref. 6:p. 218]

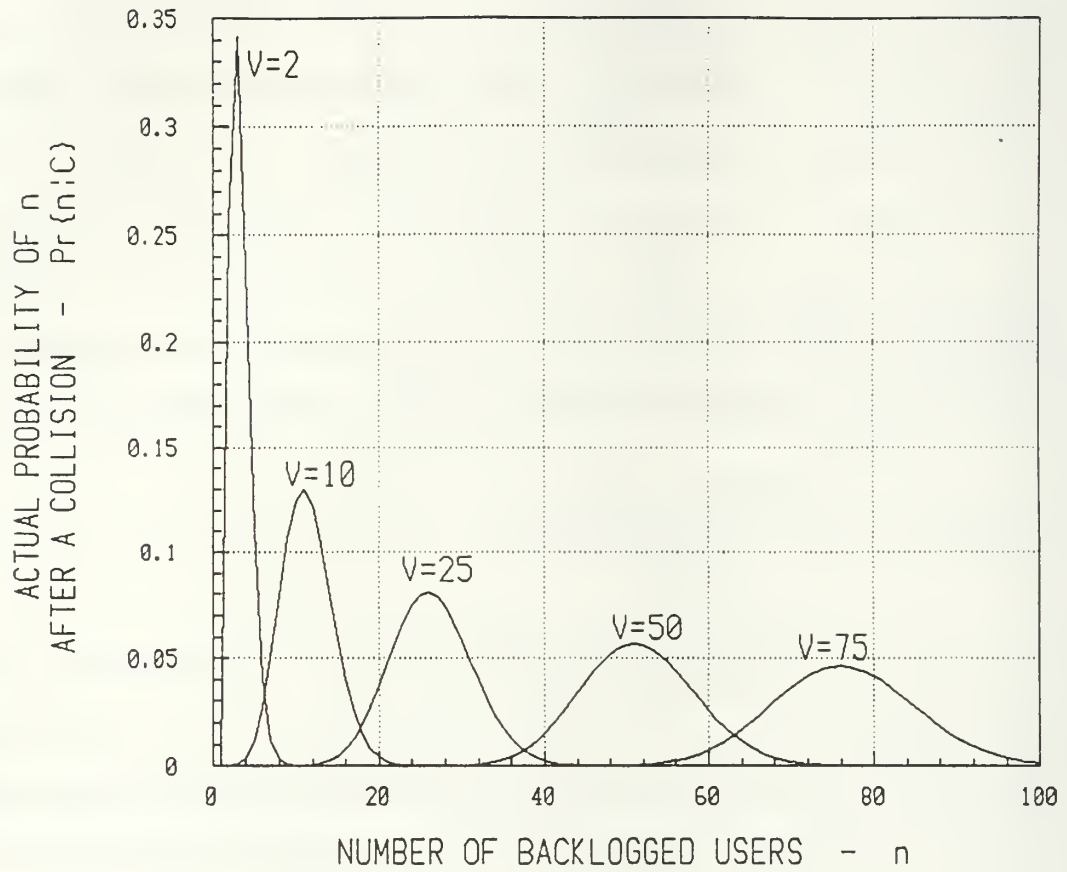


Figure 4.1: Actual Probability Distribution of the Number n of Backlogged Users After a Collision Occurs in a Conventional Stabilized ALOHA Network.

After updating the estimate v of the number of backlogged stations according to the pseudo-Bayesian method outlined above, any new packets generated during the slot must be added to v to maintain accuracy in the estimate. On the average, the channel input rate λ provides the expected number of newly generated packets in the slot.

To summarize the pseudo-Bayesian algorithm used to update the estimate of the expected number of backlogged

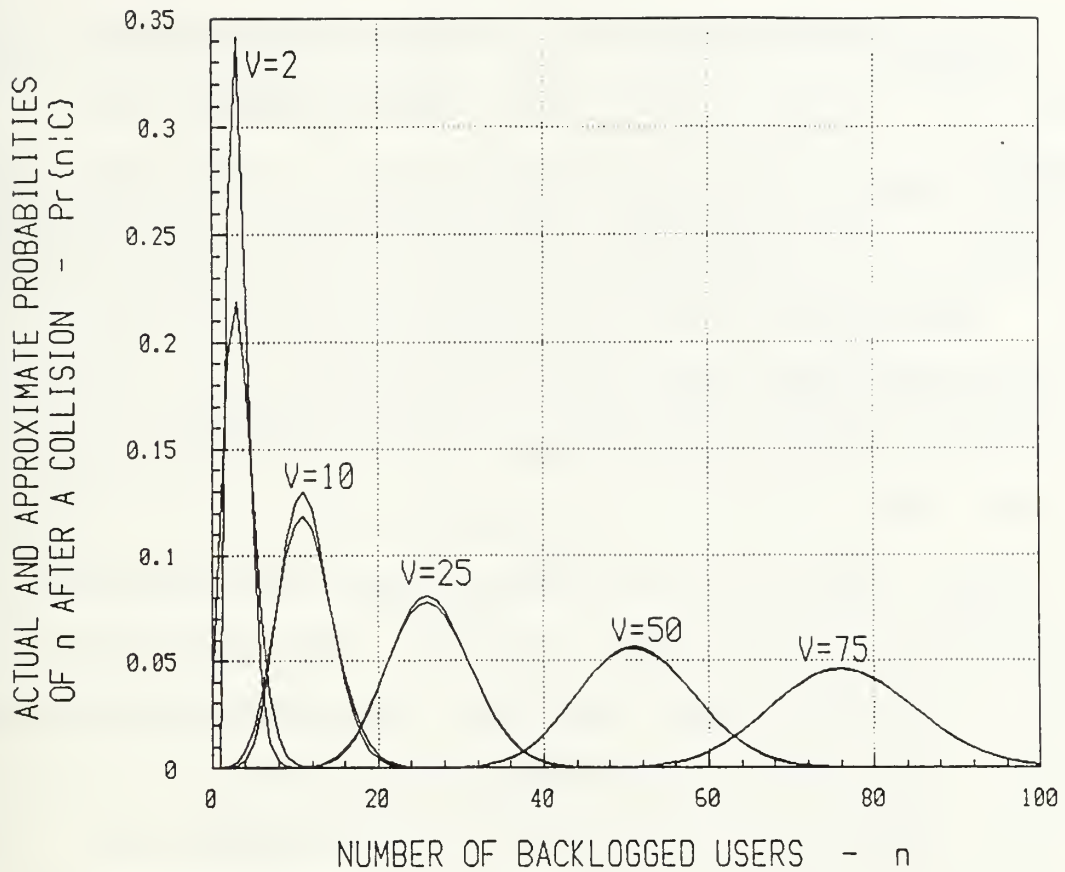


Figure 4.2: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a Conventional Stabilized ALOHA Network.

stations in the network, v_k will denote the estimate for slot k and v_{k+1} will denote the updated estimate used for the succeeding slot. The updated estimate of the expected number of backlogged stations in the network is obtained from the limited feedback provided by the system and the previous estimate of the backlog according to the following rule [Ref. 6:p. 218]

$$v_{k+1} = \begin{cases} \max(\lambda, v_k + \lambda - 1) & ; \text{ for idle or success} \\ v_k + \lambda + [\exp(1) - 2]^{-1} & ; \text{ for collision} \end{cases} \quad (4.15)$$

After determining the estimate v_{k+1} , users transmit packets ready for transmission in slot $k+1$ with probability $1/v_{k+1}$.

Experimental results obtained through simulation of the pseudo-Bayesian algorithm at the Massachusetts Institute of Technology have demonstrated that stabilization of slotted ALOHA networks can be accomplished for $\lambda \leq 1/e$, that is, for all values of the channel input rate less than the maximum throughput [Ref. 21:p. 8]. In most applications, little is known about the actual value of the channel input rate other than it satisfies the relation $\lambda \leq 1/e$. In these situations, λ is set to its maximum value of $1/e$ without adverse consequences to the algorithm. To heuristically understand why the algorithm performs as expected, consider the following. The system is characterized by the values of n and v . For large backlogs, if $n = v$, each of the backlogged packets is independently transmitted with probability $q_r = 1/n$. Therefore, the channel traffic rate G is one packet per slot and throughput is maximized. If $n > v$, the channel traffic rate will be larger than one packet per slot and collisions will occur more frequently than idle or success slots. Although n continues to increase in this case, v grows at a faster rate. The difference $n-v$ converges to

zero and throughput approaches the maximum achievable. If $n < v$, the channel traffic rate will be smaller than one packet per slot. Idle and success slots will occur more frequently than collision slots. Because of the reduction in the estimate due to idle slots, v will decrease more rapidly than n . The difference $n-v$ again converges to zero and the throughput increases towards the maximum. [Ref. 6: pp. 218-219]

B. STABILIZATION OF SLOTTED ALOHA WITH TWO POWER LEVELS

To determine the optimal broadcast probability in slotted ALOHA networks using two received signal power levels $\Omega = \{1, M\}$, the expected probability of a success slot is considered. With n backlogged stations in the network transmitting independently with probability q_r , the probability of a success slot is given by

$$\Pr\{S|n\} = \sum_{m=1}^n c(n,m) \cdot q_r^m \cdot (1-q_r)^{n-m} \cdot \Pr\{\text{capture}|m\} \quad (4.16)$$

Since the user population is treated as a group and a separate tagged packet is not considered, the probability of capture given in Chapter III must be modified slightly. Obviously from (3.4), if only one packet exists in the slot, it will be captured by the receiver and be successfully received. If more than one data packet is transmitted in a given slot, the receiver may capture one

of the transmitted packets if it has the higher power level M and all of the interfering packets have the lower power level 1. Since the maximum number of interfering packets that can be tolerated by the packet with the higher power level is $N = \lfloor M/\gamma_0 \rfloor$, the number of packets in the slot must be less than or equal to $N + 1 = \lfloor M/\gamma_0 \rfloor + 1$. Therefore, the probability that one packet captures the receiver and is accurately received is modified to

$$\Pr\{\text{capture}|n\} = \begin{cases} 1 & ; m = 1 \\ \frac{m}{2^m} & ; 2 \leq m \leq N+1 \\ 0 & ; m > N+1 \end{cases} \quad (4.17)$$

Therefore,

$$\begin{aligned} \Pr\{S|n\} &= n \cdot q_r \cdot (1-q_r)^{n-1} \\ &+ \sum_{m=2}^{N+1} c(n,m) \cdot q_r^m \cdot (1-q_r)^{n-m} \cdot \frac{m}{2^m} \end{aligned} \quad (4.18)$$

where the definition of $c(n,m)$ in (2.8) insures that the terms under the summation are zero if $n < N+1$. Averaging (4.18) over all possible values of the number of backlogged stations that can possibly result in a success slot yields the average probability of a success slot; that is,

$$\begin{aligned}
\Pr\{S\} &= \sum_{n=1}^{\infty} \Pr\{S|n\} \cdot \Pr\{n\} \\
&= \sum_{n=1}^{\infty} \frac{\exp(-v) \cdot v^n}{n!} \cdot \left[n \cdot q_r \cdot (1-q_r)^{n-1} + \sum_{m=2}^{N+1} c(n,m) \cdot q_r^m \cdot (1-q_r)^{n-m} \cdot \frac{m}{2^m} \right] \\
&= v \cdot q_r \cdot \exp(-v \cdot q_r) \cdot \left[1 + \sum_{m=2}^{N+1} \frac{(v \cdot q_r)^{m-1}}{2^m \cdot (m-1)!} \right] \quad (4.19)
\end{aligned}$$

Comparison of (4.19) with the expression for the throughput of a slotted ALOHA network given in (3.9) demonstrates that the probability of a success slot will be maximized if $v \cdot q_r$ is set equal to the value of the channel traffic rate giving the maximum throughput achievable, denoted by G_S . Therefore, the broadcast probability q_r utilized in slotted ALOHA networks with two received signal power levels is given by

$$q_r = \min \left\{ 1, \frac{G_S}{v} \right\} \quad (4.20)$$

where G_S can be found by differentiating (3.9).

Assuming that the number n of backlogged stations in the network at a given time is a Poisson random variable with mean $v > 1$ and that each transmits its data packet with probability $q_r = G_S/v$, application of Bayes Rule to (4.1), (4.18) and (4.19) yields the probability distribution of n conditioned on a success slot given by

$$\Pr\{n|S\} = \frac{\Pr\{S|n\} \cdot \Pr\{n\}}{\Pr\{S\}}$$

$$= \frac{\left[\frac{\exp(-v+G_S)}{(n-1)!} \cdot (v-G_S)^{n-1} \cdot \left[1 + \frac{1}{2 \cdot n} \cdot \sum_{m=2}^{N+1} c(n,m) \cdot \left(\frac{G_S}{2} \right)^{m-1} \cdot (v-G_S)^{1-m \cdot m} \right] \right]}{\left[1 + \frac{1}{2 \cdot n} \cdot \sum_{m=2}^{N+1} \left(\frac{G_S}{2} \right)^{m-1} \cdot \frac{1}{(m-1)!} \right]}$$

(4.21)

This probability distribution is shown in Figure 4.3 for various values of v and $\Omega = \{1, 6\gamma_0\}$. Similar results are obtained for other values of N . To preserve the Poisson distribution of the number of backlogged packets, $\Pr\{n|S\}$ given by (4.21) can be approximated by

$$\Pr\{n|S\} = \frac{\exp(-v+1) \cdot (v-1)^{n-1}}{(n-1)!} \quad (4.22)$$

Figure 4.4 compares the actual distribution of the number of backlogged stations with the approximating Poisson distribution. As can be observed, the approximation is exceptionally good. Similar results are obtained for other values of N . Therefore, when a success slot occurs, the estimate of the expected number v of backlogged users is decremented by one.

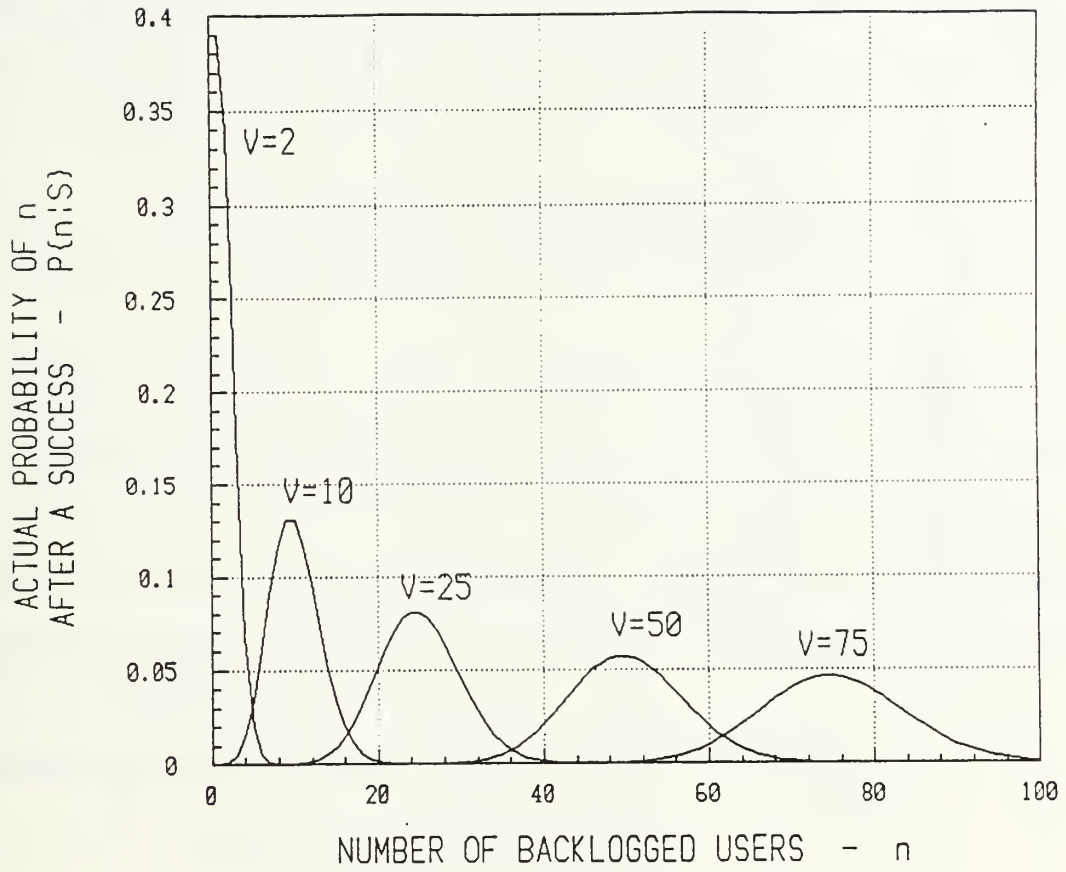


Figure 4.3: Actual Probability Distribution of the Number n of Backlogged Users After a Success Slot Occurs in a Stabilized ALOHA Network Utilizing Two Power Levels. $\Omega = \{1, 6\gamma_0\}$.

When n backlogged stations exist in the network and independently transmit their data packets in each slot with probability $q_r = G_S/v$, the probability that an idle slot will occur is given by

$$\Pr\{I|n\} = \left[1 - \frac{G_S}{v}\right]^n \quad (4.23)$$

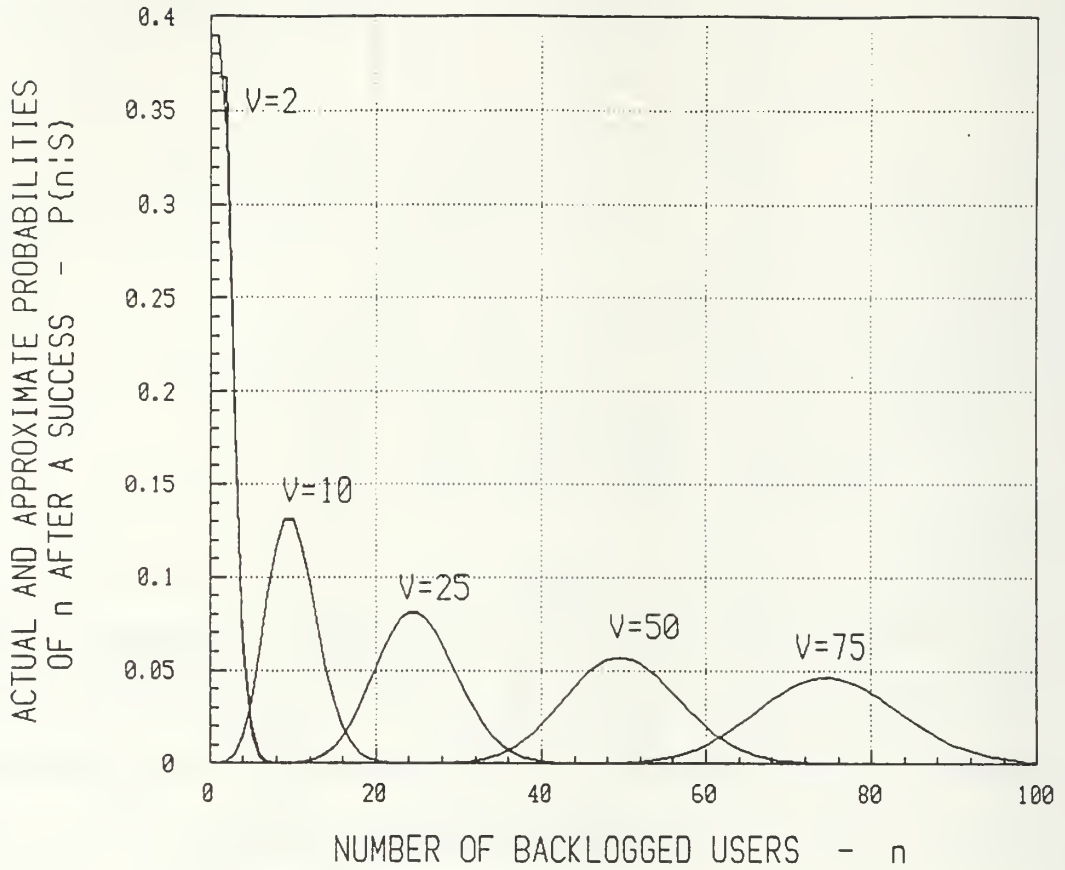


Figure 4.4: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Success Slot Occurs in a Stabilized ALOHA Network Utilizing Two Power Levels. $\Omega = \{1, 6\gamma_0\}$.

Averaging over the number of backlogged stations yields the expected probability of an idle slot; that is,

$$\begin{aligned}
 \Pr\{I\} &= \sum_{n=0}^{\infty} \Pr\{I|n\} \cdot \Pr\{n\} \\
 &= \sum_{n=0}^{\infty} \left[1 - \frac{G_S}{v} \right]^n \cdot \frac{\exp(-v) \cdot v^n}{n!} \\
 &= \exp(-G_S)
 \end{aligned} \tag{4.24}$$

Bayes Rule, applied to (4.1), (4.23) and (4.24), gives the probability distribution of the number of backlogged users after an idle slot as

$$\Pr\{n|I\} = \frac{\Pr\{I|n\} \cdot \Pr\{n\}}{\Pr\{I\}} = \frac{\exp(-v+G_S) \cdot (v-G_S)^n}{n!} \quad (4.25)$$

Therefore, when an idle slot occurs, the number of backlogged stations has a Poisson distribution with mean $\max\{v-G_S, 0\}$. In the event of an idle slot, users reduce their estimate of the expected number of backlogged stations by G_S , unless v is already less than G_S in which case v is set to zero.

The expected probability of a collision slot is

$$\begin{aligned} \Pr\{C\} &= 1 - \Pr\{I\} - \Pr\{S\} \\ &= 1 - \exp(-G_S) \cdot \left[1 + G_S \cdot \left[1 + \frac{1}{2} \cdot \sum_{m=2}^{N+1} \left(\frac{G_S}{2} \right)^{m-1} \cdot \frac{1}{(m-1)!} \right] \right] \end{aligned} \quad (4.26)$$

Use of Bayes Rule in this situation leads to a complicated expression for the distribution of the backlogged stations after a collision slot so an alternate method using the total probability theorem [Ref. 19:p. 89]

$$\Pr\{n\} = \Pr\{n|I\} \cdot \Pr\{I\} + \Pr\{n|S\} \cdot \Pr\{S\} + \Pr\{n|C\} \cdot \Pr\{C\} \quad (4.27)$$

will be used. Thus, the distribution of the backlogged users after a collision slot is

$$\begin{aligned}
\Pr\{n|C\} &= \frac{\Pr\{n\} - \Pr\{n|I\} \cdot \Pr\{I\} - \Pr\{n|S\} \cdot \Pr\{S\}}{\Pr\{C\}} \\
&= \frac{\frac{\exp(-v)}{n!} \cdot \left[v^{n-(v-G_S)} \cdot \left[1 + \sum_{m=2}^{N+1} c(n,m) \cdot \left(\frac{G_S}{2}\right)^{m-1} \cdot (v-G_S)^{1-m} \right] \right]}{1 - \exp(-G_S) \cdot \left[1 + G_S \cdot \left[1 + \frac{1}{2} \cdot \sum_{m=2}^{N+1} \left(\frac{G_S}{2}\right)^{m-1} \cdot \frac{1}{(m-1)!} \right] \right]}
\end{aligned} \tag{4.28}$$

This distribution is shown in Figure 4.5. As in the stabilization of conventional slotted ALOHA systems, this function can be approximated by a Poisson distribution with mean $v + [\exp(1) - 2]^{-1}$; that is,

$$\Pr\{n|C\} = \frac{\exp[v + (\exp(1) - 2)^{-1}]}{n!} \cdot [v + (\exp(1) - 2)^{-1}]^n \tag{4.29}$$

A comparison of the distributions of the number of backlogged stations in the network given by (4.28) and (4.29) is shown in Figure 4.6. As shown in Figure 4.6, use of the Poisson approximating distribution function provides a reasonable estimation of the actual distribution. Therefore, the estimate of the expected number of backlogged users is incremented by $[\exp(1) - 2]^{-1}$ when feedback indicates a collision occurred.

To account for newly generated packets in the estimate, the estimate v is increased by the value of the channel

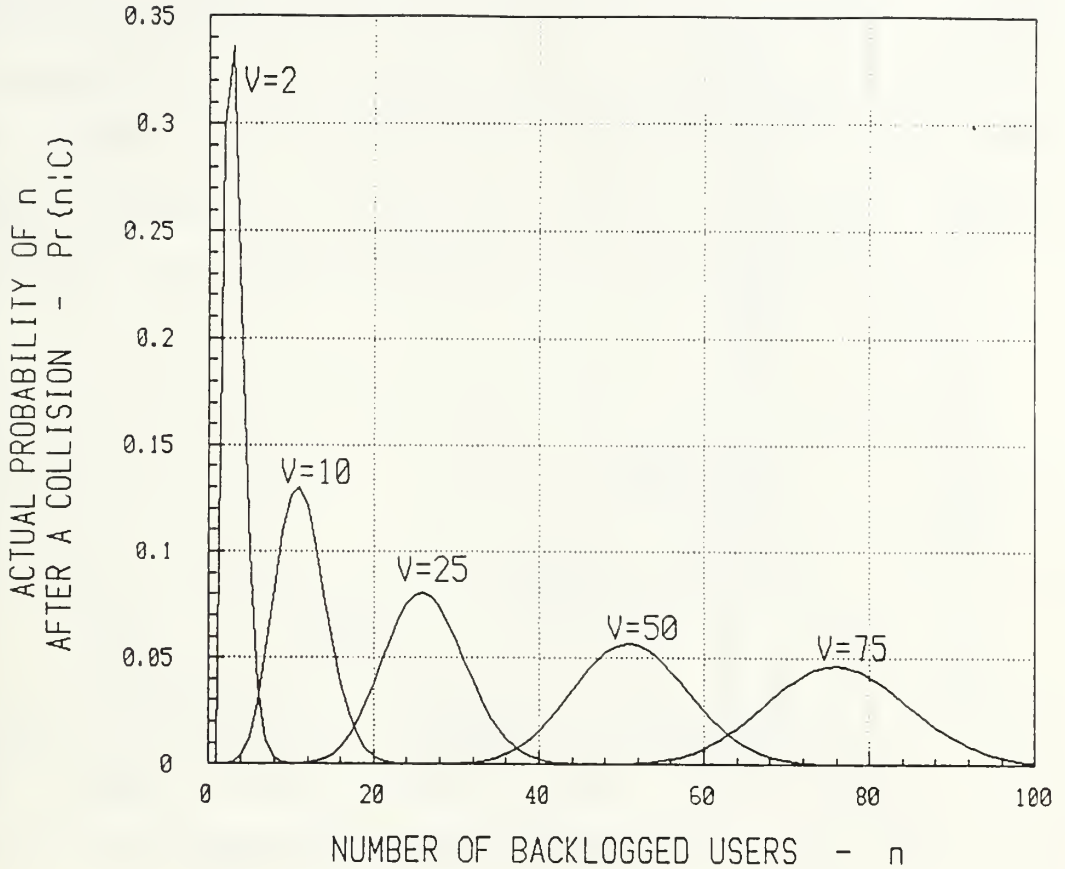


Figure 4.5: Actual Probability Distribution of the Number n of Backlogged Users After a Collision Occurs in a Stabilized ALOHA Network Utilizing Two Power Levels. $\Omega = \{1, 6\gamma_0\}$.

input rate λ . Since this is a Poisson process, the Poisson approximation of the channel traffic rate is preserved by the estimate.

To summarize the pseudo-Bayesian algorithm used to stabilize slotted ALOHA networks utilizing two received signal power levels, v_k will denote the estimate of the expected number of backlogged stations in the network and v_{k+1} will denote the updated estimate used to determine

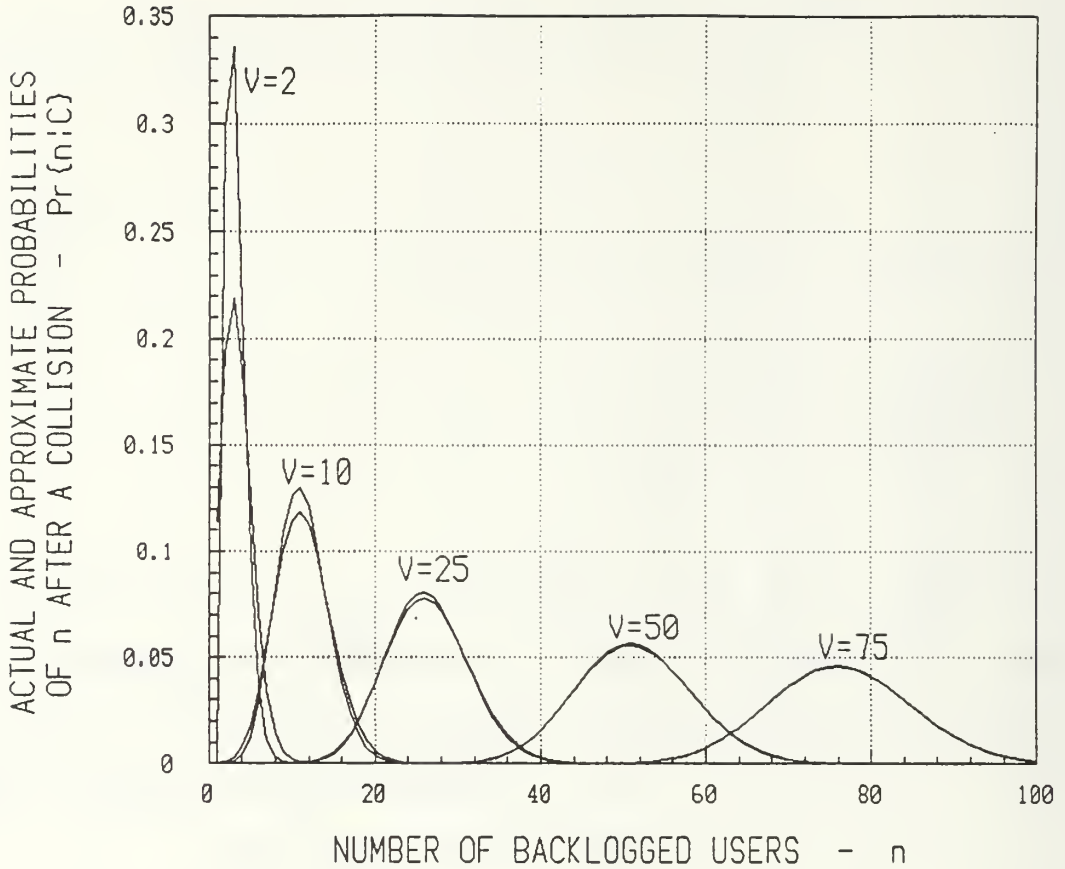


Figure 4.6: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a Stabilized ALOHA Network Utilizing Two Power Levels. $\Omega = \{1, 6\gamma_0\}$.

the broadcast probability for the succeeding slot. The broadcast probability q_r for slot $k+1$ is

$$q_r = \min\left\{1, \frac{G_S}{v_{k+1}}\right\} \quad (4.30)$$

where G_S denotes the value of the channel traffic rate that yields the maximum achievable throughput $S_{\max}$ in the

unstabilized ALOHA system using $\Omega = \{1, M=\lceil N \cdot \gamma_0 \rceil\}$. G_S can be determined by differentiating (3.9) with respect to G and finding the root of the resulting equation. The updated estimate is obtained from the limited feedback provided by the system according to the following rule

$$v_{k+1} = \begin{cases} \max\{\lambda, v_k + \lambda - G_S\} & ; \text{ for idle slot} \\ \max\{\lambda, v_k + \lambda - 1\} & ; \text{ for success slot} \\ v_k + \lambda + [\exp(1) - 2]^{-1} & ; \text{ for collision slot} \end{cases} \quad (4.31)$$

The pseudo-Bayesian algorithm, as adapted to slotted ALOHA systems using two power levels, should perform in a similar manner to the conventional ALOHA case. Given the set $\Omega = \{1, M\}$, the system should stabilize at the appropriate maximum achievable throughput $S_{\max}$ shown in Figure 3.4. Since the throughput is significantly higher than that achieved in conventional slotted ALOHA networks, stabilization can be accomplished at channel input rates larger than $1/e$, although it is expected that λ must remain less than $S_{\max}$.

Whether stabilizing conventional or multiple power level ALOHA networks, the pseudo-Bayesian algorithm allows growth of the user population without any major complications. Since all users maintain the same estimate of the expected number of backlogged users, inclusion of

this information in the required overhead information for each transmitted packet would allow users new to the network to synchronize quickly. Alternatively, the receiver could provide its computed value of the estimate in the feedback for success slots. [Ref. 21:p. 8]

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The theory needed to analyze the throughput achieved in ALOHA networks using multiple received signal power levels in nonfading environments has been developed. In the models considered, priority classes among the network users are avoided since each station chooses a received power level at random from a given set for each data packet transmitted. Realistic capture thresholds that produce usable probabilities of error in the data packets have been incorporated into the analysis.

The use of a set of M equally spaced received power levels in slotted ALOHA networks offers no great advantage over conventional ALOHA when realistic capture thresholds between 6 dB and 12 dB are considered. In both slotted and unslotted protocols, the throughput and average packet transfer delay is greatly improved when the beneficial power capture effects are created by utilizing only two received power levels, with magnitudes selected on the basis of the capture threshold of the receiver and the number of interfering packets encountered at the receiver. Although the inherent instability of the ALOHA random multiple access protocol persists and the network must still operate at throughput rates well below the maximum,

use of two received signal power levels allows operation at higher throughput rates with greater margin for peak traffic demands.

Very little is known about stabilizing unslotted ALOHA networks. Slotted ALOHA networks, on the other hand, can be stabilized such that the system operates at the maximum throughput achievable. The pseudo-Bayesian stabilization algorithm used in conventional slotted ALOHA networks requires little theoretical modification for use in two power level slotted ALOHA systems.

The throughput obtained when multiple power levels are employed in fading environments or under other random multiple access protocols is a natural extension to the results presented here. These topics were investigated as part of the research supporting this thesis. References 11 and 12 demonstrate significant improvement in the performance of ALOHA networks utilizing multiple power levels to create the power capture effects in a fading environment. Reference 22 shows that the carrier sense multiple access (CSMA) and carrier sense multiple access with collision detection (CSMA/CD) protocols, which are derivatives of the ALOHA protocol, experience gains in the throughput achieved with the use of multiple power levels, but the improvement is less significant than the gains obtained in slotted and unslotted ALOHA.

B. RECOMMENDATIONS

Reference 11 presents a short discussion on the throughput obtained when a set of three power levels with unequally spaced magnitudes is utilized to create power capture effects in a slotted ALOHA network. The results show moderate improvement over the two power level model. A thorough investigation of ALOHA networks utilizing a set of M unequally spaced received signal power levels should be completed. Topics such as the optimum number of power levels, their magnitude and stabilization should be included in this investigation.

The pseudo-Bayesian stabilization algorithm theoretically developed for two power level slotted ALOHA should perform well in maintaining the channel traffic rate that yields the maximum throughput achievable. To confirm this hypothesis, a network implementing the algorithm should be simulated. The effects of fading on the pseudo-Bayesian algorithm should also be studied.

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